PRESTRESSED CONCRETE

FIFTH EDITION

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Prestressed Concrete

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Preface

Preface to the Fifth Edition

The prime purpose of this new edition of Prestressed Concrete is to take account of important changes that have been made in the new edition of the Australian Standard for Concrete Structures, which appeared in 2018 and Amendment 2 in 2021. Changes include modifications to the rectangular stress-block parameters for flexural strength calculations, modifications to the design clauses for shear and torsion, and changes in the safety coefficients for ultimate strength design which result in a slight reduction in the overall safety margins for design. We have also taken the opportunity to correct minor errors and to rearrange the material in Appendix B, which deals with the effects of creep and shrinkage in prestressed flexural members, to make it more directly applicable to the design process.

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Adelaide	Sydney	Bendigo	Melbourne

November, 2021

Preface to the Fourth Edition

Following the death of Ken Faulkes, who was an original co-author of previous editions of this book, Rebecca Gravina has joined the team of co-authors in the preparation of the fourth edition of *Prestressed Concrete.* In this new edition we have taken the opportunity to make corrections to the text and to extend and improve the treatment of creep and shrinkage effects in Appendix B.

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December, 2016

Preface to the Third Edition

When the first edition of this book appeared, more than thirty years ago, prestressed concrete and reinforced concrete were considered to be separate and distinct materials of construction. In Australia, different design standards had to be used for the two materials. At that time, prestressed concrete was designed to remain uncracked under full working load, using elastic analysis and allowable stress concepts. The apparent gulf between reinforced concrete and prestressed concrete was exaggerated by severe restrictions that were placed on the use of partial prestressing by the then-current prestressed concrete standard, AS 1481. One of the main aims of the first edition of this book was to present a rational and unified approach to the analysis and design of prestressed concrete, and hence to encourage designers to choose freely from the full range of design options, including reinforced concrete and any appropriate level of prestress. A further aim of the book was to present rational ways of selecting prestress levels that would optimise service load behaviour and economy. This aim was reflected in its full title: *Prestressed Concrete: with emphasis on partial prestressing*[1].

Today, the Australian Concrete Structures Standard, AS 3600, adopts a unified, performance-based approach to design for all reinforced concrete structures and members, irrespective of whether or not prestress is used. This integrated approach has removed unnecessary restrictions on design and has provided significant advantages to innovative designers. This in turn has resulted in partial prestressing becoming the design norm, and the term prestressed concrete now means, simply, structural concrete with prestress.

The world-wide developments that occurred in the field of concrete structures also made possible a more elegant, unified treatment of the underlying theory. Structural concrete members may contain any mix of reinforcing and prestressing steel, with reinforced concrete members being at one end of this continuum. Such a unified approach to analysis and design was adopted in a previous text book, titled *Concrete Structures*[2].

Nevertheless, when we consider the processes of teaching and learning today, we find that many students and lecturers prefer to deal first with the basics of reinforced concrete before going on to consider the concepts of prestressed concrete. Furthermore, it is now usual for many undergraduate course programs in structural engineering to cover reinforced concrete but not prestressed concrete. Although used widely in practice, prestressed concrete has become an option for undergraduate students, and hence a subject to be studied by postgraduates and practising engineers. For these reasons, we have

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[1] Warner, R. F. and Faulkes, K. A., 1979, Prestressed Concrete: with emphasis on partial prestressing, Pitman Publishing, Melbourne.

[2] Warner, R. F., Rangan, B. V., Hall, A. S. and Faulkes, K. A., 1998, Concrete Structures, Longman-Cheshire, Melbourne.

chosen to prepare a new edition of *Prestressed Concrete*, rather than *Concrete Structures*, in order to provide an up-to-date treatment of prestressed concrete. We are in fact reverting to the approach we originally adopted when the first editions of *Reinforced Concrete*[3] and *Prestressed Concrete*[1] appeared in 1977 and 1979.

The present book is thus intended as a companion to *Reinforced Concrete Basics*[4], which appeared recently, in 2007, and provides a thorough treatment of the analysis and design of reinforced concrete structures and members. This third edition of *Prestressed Concrete* maintains a basic and rational approach to the analysis and design of prestressed concrete. It builds on, but integrates with, the ideas of reinforced concrete. For detailed design, both books refer to the requirements of the new edition of the Australian Concrete Standard, AS 3600-2009.

The sequencing of material in this new edition follows that of the second edition. Chapter 1 presents a simple, introductory, non-mathematical overview of the field of prestressed concrete, including both analysis and design, and introduces the important design concepts of equivalent loads and load balancing. Chapters 2 and 3 provide additional preliminary information on material properties and codified design procedures. Chapters 4 to 6 deal with the behaviour and design of flexural members, while Chapters 7 to 9 fill in important information on non-flexural behaviour, namely shear, torsion, anchorage and losses. The final Chapters, 10 to 12, concentrate on design aspects for determinate and indeterminate beams and floor slabs.

Chapters 1, 4 to 6 and 10 together provide the basis for an introductory course on the analysis and design of prestressed concrete members. Some additional material from Chapters 7 to 9 is however needed to round out the treatment to introduce the topics of shear, losses and anchorage.

The contents of the book have been extensively revised, updated and rewritten to take account of the many developments in theory and practice that have occurred in the intervening years since the second edition. The treatment of creep and shrinkage effects in prestressed concrete flexural members, in Chapters 4 and 5 and Appendix B, has been simplified by using an approach

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[3] Warner, R. F., Rangan, B. V. and Hall, A. S., 1977, Reinforced Concrete, Pitman Publishing, Melbourne.

[4] Foster, S. J., Kilpatrick, A. E. and Warner, R. F., 2021, Reinforced Concrete Basics, 3rd Ed., Pearson Education Australia, Melbourne.

that is based on the fundamental structural concepts of equilibrium, compatibility and elastic behaviour. The use of more complex visco-elastic analyses has thus been avoided. A greater emphasis has been placed on strut-and-tie modelling in Chapters 7 and 8 to deal with shear, torsion, and anchorage. This reflects recent developments in our knowledge and understanding of this important design tool.

The authors are deeply indebted to their friend and colleague Andrew Kilpatrick. It is only through his unstinting help that we have been able to complete the text.

It is with great sadness that we acknowledge the death of our long-time friend and colleague Professor Ken Faulkes. Ken was working with us on the final chapters of the book at the time of his death. His contributions have been invaluable. His vast knowledge and his experience, based on more than 50 years of research and practice in the field, are reflected throughout this text.

Robert Warner   Stephen Foster
1 June 2013

Notation

A_{c} = area of concrete

A_{eq} = combined equivalent area of steel and tendon

A_{g} = gross area of a non-transformed cross-section

A_{p} = area of prestressing steel

A_{pt} = area of prestressing steel on the tension side of the neutral axis at M_{u}

A_{s} = area of reinforcement

A_{sc} = area of longitudinal compressive reinforcement

A_{st} = area of longitudinal tensile reinforcement

A_{sv} = area of all legs of vertical shear reinforcement that cross a shear crack

$A_{\mathrm{sv.min}}$ = minimum required area of shear reinforcement in a beam

a = a coefficient
= shear span
= clear distance between reinforcing bars
= distance between points of contraflexure

a_{sup} = length of a support for a flexural member

a_{v} = distance from the section at which shear is being considered to the face of the nearest support

b = width of a rectangular cross-section

b_{ef} = effective width of the flange of a T- or L-section

b_{v} = effective width of the web of a beam for shear

b_{w} = web width of a T- or L-section

C = compressive force

C_{c} = compressive force in the concrete of a cross-section

C_{f} = compressive force in the flange outstands of a T- or L-section

C_{sc} = compressive force in the steel reinforcement of a cross-section

C_{w} = compressive force in the web of a T- or L-section

C^* = compressive force at the strength limit state

c = cover of concrete to a reinforcing bar

D = overall depth of a member

D_{s} = overall depth of a slab

d = effective depth, from the extreme fibre in compression to the resultant of the steel forces (T_s and T_p), on the tension side of a cross-section.

d_c = depth from the extreme fibre in compression to the concrete compressive stress resultant

d_{eq} = depth in section of equivalent area A_{eq}

d_n = depth to the neutral axis in a cross-section in bending

= depth to the neutral axis of strain, if inelastic strains are present

d_{ne} = depth to the neutral axis of stress if inelastic strains are present

d_o = depth from the extreme fibre in compression to the centroid of the outermost layer of longitudinal tensile reinforcement, but not less than $0.8D$

= diameter of a reinforcing bar

d_{om} = mean value of d_o around the punching shear perimeter u_e

d_p = depth from the extreme compressive fibre to the prestressing steel

d_s = depth from the extreme compressive fibre to the reinforcing steel

e = eccentricity of prestress force P

e_{eq} = eccentricity of equivalent area A_{eq} in section

E_c = mean secant modulus of elasticity of in situ concrete measured at an axial stress of 45 per cent of the peak stress

E_{co} = modulus of elasticity of concrete at time t_o

E_c^* = modulus of elasticity of concrete at time t^*

E_d = design action effect

E_{eq} = modulus of elasticity of equivalent area steel A_{eq}

E_P = mean modulus of elasticity of prestressing steel

E_R = reduced elastic modulus for concrete, in creep calculations

$E_{R\chi}$ = age-adjusted effective modulus for concrete, used in creep calculations

E_s = mean modulus of elasticity of the steel reinforcement

e = eccentricity of a prestressing force from the section centroid

F = force

f_{cm} = mean compressive strength of a concrete cylinder

f_{cmi} = mean compressive strength of in situ concrete

f_p = sustained stress in prestressing tendon

f_{cmi} = mean compressive strength of in situ concrete

f_{ct} = tensile strength of concrete

$f_{ct.f}$ = mean concrete tensile strength obtained from a flexure (modulus of rupture) test

f_{cv} = concrete punching shear stress capacity

f_{pb} = characteristic minimum breaking stress of prestressing steel
f_{s} = stress limit in reinforcing steel to control serviceability cracking
f_{su} = tensile strength of steel reinforcement (ultimate)
f_{sy} = characteristic yield strength of steel reinforcement
$f_{\mathrm{sy.f}}$ = characteristic yield strength of steel reinforcement used as fitments (stirrups)
f'_{c} = characteristic compressive strength of a concrete cylinder at age 28 days
f'_{cp} = characteristic compressive strength of concrete at transfer
f'_{ct} = characteristic uniaxial tensile strength of concrete
$f'_{\mathrm{ct.f}}$ = characteristic value of $f_{\mathrm{ct.f}}$
G = service permanent action (dead load)
h = midpoint sag of a parabola
h_{d} = width of a bearing plate
h_{s} = depth of a bearing plate
h_x = midpoint sag of a parabolic cable in a slab in the x-direction
h_y = midpoint sag of a parabolic cable in a slab in the y-direction
I_{cr} = second moment of area of a cracked, transformed cross-section
I_{cs} = average second moment of area of the column strip in a slab
I_{wb} = average second moment of area of the wide-beam portion of a slab
I_{ef} = effective second moment of area
$I_{\mathrm{ef.av}}$ = weighted average effective second moment of area for a flexural member
I_{g} = second moment of area of the gross concrete cross-section
k = a factor
k_{cs} = long-term deflection factor
k_{u} = neutral axis depth parameter for a cross-section, d_{n}/d at M_{u}
k_{uo} = neutral axis depth parameter in a cross-section, $d_{\mathrm{n}}/d_{\mathrm{o}}$, when M_{uo} is acting
$k_1, k_2, k_3, k_4, k_5, k_6$
= deflection coefficients or modifying factors
$E^*_{\mathrm{R}\chi}$ = age-adjusted effective modulus at time t^*
j = time in days after prestressing
L = centre-to-centre distance of the supports for a flexural member
L_{n} = clear span, face-to-face of supports

L_{pa} = length of beam from the jack to the point at which friction loss is being calculated

L_x, L_y = shorter and longer spans, respectively, of a slab supported on all four sides

M = bending moment

M_{cr} = bending moment that causes a cross-section to begin cracking (including shrinkage)

M_{cro} = bending moment that causes a cross-section to begin cracking (excluding shrinkage)

M_{dec} = decompression moment (zero bottom fibre stress)

M_G = bending moment due to self-weight

M_{GQ} = bending moment due to self-weight plus live load

M_L, M_M, M_R = bending moments at the left, middle and right ends of a beam or slab

M_o = bending moment corresponding to zero curvature

= decompression moment

M_p = section moment due to prestress (= $P.e$ in a determinate member)

= $M_1 + M_2$; total moment in a section in an indeterminate member due to prestress

M_G = bending moment due to dead load

M_Q = bending moment due to live load

M_u = ultimate bending capacity (strength) of a cross-section

M_{uo} = value of M_u in the absence of an axial force

M_y = bending moment at which the tensile reinforcement yields

M^* = bending moment at a cross-section due to the design load for the strength limit state

M^*_{max} = maximum value of M^* in a flexural member

M^*_v = bending moment to be transmitted from a slab to a column at the strength limit state

M_1 = primary moment due to prestress in a section of an indeterminate beam

M_2 = secondary (hyperstatic) moment due to prestress in a section of an indeterminate beam

N = axial force

n = modular ratio

P = prestressing force

= force acting normal to a cross-section

P_e = effective prestressing force

P_i = initial prestressing force, prior to time-dependent losses

P_T = total prestressing force in a slab per panel in one direction

P_u = breaking load of prestressing tendon

= capacity of prestressed section against failure at transfer

P_v = vertical component of prestressing force in a tendon

P_x = prestressing force per unit width of slab in x-direction

P_y = prestressing force per unit width of slab in y-direction

P^* = concentrated load for the strength limit state

p_cw = ratio of the area of compressive reinforcement to the area of the web, $A_\mathrm{sc}/(b_\mathrm{w}d)$

p_w = ratio of the area of the tensile reinforcement to the area of the web, $(A_\mathrm{st}+A_\mathrm{pt})/b_\mathrm{w}d$

Q = service imposed action (live load)

R = resultant force

= design relaxation of prestressing tendon

R_b = basic relaxation of prestressing tendon

R_d = design capacity

R_u = nominal ultimate capacity

$R_\mathrm{u.sys}$ = mean value of the calculated capacity of a structural system

S = vertical component of prestressing force at an anchorage

s = spacing of fitments (stirrups) along a beam

T = tensile force

= average annual temperature

T_p = force in a prestressing tendon

T_s = force in the longitudinal tensile reinforcement

t = thickness (depth) of a slab

= thickness of the compression flange of a section

= time

t_h = hypothetical thickness of a member

t^* = time infinity

u = perimeter around which punching shear occurs

u_e = that part of the perimeter of a member that is exposed to a drying environment, plus half the perimeter of any internal voids

V = shear force

V_1 = shear force carried by the concrete at web-shear cracking

V_dec = shear force in section at decompression moment

V_{p} = vertical component of prestressing force

V_{s} = tensile force in a vertical stirrup

V_{u} = flexural shear capacity of a beam containing shear reinforcement

V_{uc} = ultimate flexural shear capacity of a beam without shear reinforcement

= inclined shear cracking load

$V_{\mathrm{u.max}}$ = maximum flexural shear capacity of a beam

$V_{\mathrm{u.min}}$ = flexural shear capacity of a beam containing minimum shear reinforcement

V_{uo} = punching shear capacity of a slab without moment transfer

V_{us} = contribution by the shear reinforcement to the flexural shear capacity of a beam

V^* = flexural shear force at a cross-section due to the design load for the strength limit state

V^*_{max} = maximum value of V^* in a flexural member

W = concentrated load

W_{p} = equivalent concentrated load exerted by a tendon at a kink

W^* = concentrated load for the strength limit state

w = uniformly distributed load kN/m

w_{b} = uniformly distributed load to be balanced

w_{G} = uniformly distributed dead load

w_{l} = uniformly distributed long-term serviceability load, $w_{\mathrm{G}} + \psi_{\mathrm{l}} w_{\mathrm{Q}}$

w_{p} = distributed equivalent load from a curved prestressing tendon

w_{px} = distributed equivalent load from a curved x-direction prestressing tendon

w_{py} = distributed equivalent load from a curved y-direction prestressing tendon

w_{Q} = uniformly distributed service live load

w_{s} = uniformly distributed short-term serviceability load $w_{\mathrm{G}} + \psi_{\mathrm{s}} w_{\mathrm{Q}}$

w^* = uniformly distributed design load for the strength limit state

y_{b} = distance from the neutral axis to the bottom fibre of a cross-section

y = distance from the bottom fibre in an uncracked (gross) cross-section to its centroidal axis

Z = elastic section modulus of a cross-section

z = lever arm (distance) between the forces C and T of a cross-section in bending

α = an angle

= reduction factor for steel strain increment in unbonded tendons

α_1, α_2 = compressive stress-block factors

= non-dimensional parameters used to determine creep and shrinkage curvatures

α_{tot} = total angular deviation of prestressing cable over length L_{pa}

β = beam section shear capacity parameter, $\beta_1\beta_2\beta_3$

= friction coefficient

β_x, β_y = coefficients for bending moments in slabs supported on four sides

β_1, β_2, β_3

= parameters for the shear capacity of a beam cross-section

γ = compressive stress block factor

= unit weight of a material

= a coefficient

γ_c, γ_{rc} = unit weight of reinforced concrete

Δ = deflection of a member in bending

Δ_G = deflection due to self-weight

Δ_P = deflection due to prestress

Δ_{PG} = deflection due to self-weight plus prestress

Δ_s = short-term deflection due to the short-term serviceability load, $G + \psi_s Q$

$\Delta_{s.inc}$ = difference between the short-term deflections due to the loads $G + \psi_s Q$ and G (incremental deflection)

$\Delta_{s.sus}$ = short-term deflection due to the sustained serviceability load, $G + \psi_l Q$

Δ_{tot} = total long-term deflection

$\Delta_{tot.inc}$ = total long-term (incremental) deflection after the attachment of deflection-sensitive partitions or finishes

ΔX = an increment in force

ΔX_c = increment in force in the concrete

$\Delta X_{c.c}$ = increment in force in the concrete due to creep

$\Delta X^i_{c.c}$ = increment in force in the concrete, due to creep, applied at time t_i

$\Delta X^*_{c.c}$ = increment in force in the concrete, due to creep, applied at time t^*

$\Delta X^*_{c.sh}$ = increment in force in the concrete, due to shrinkage, at time t^*

ΔX_p = increment in force in the tendon

$\Delta\varepsilon$ = increment in strain

$\Delta\varepsilon_{cc}$ = increment in creep strain in the concrete

$\Delta\varepsilon_{cc.a}$ = increment in creep strain in the concrete at fibre a

$\Delta\varepsilon^{i}_{\mathrm{cc.a}}$ = increment in creep strain in the concrete at fibre a, applied at time t_{i}
$\Delta\varepsilon^{*}_{\mathrm{cc.a}}$ = increment in creep strain in the concrete at fibre a, applied at time t^{*}
$\Delta\varepsilon_{\mathrm{ce}}$ = increment in elastic strain in the concrete
$\Delta\varepsilon_{\mathrm{p}}$ = increment in strain in the tendon (all tendon strains are elastic)
$\Delta\varepsilon_{\mathrm{cc.p}}$ = increment in creep strain in the concrete at the level of the tendon
$\Delta\varepsilon_{\mathrm{p.sh}}$ = increment in strain in the tendon due to concrete shrinkage
$\Delta\kappa^{*}_{\mathrm{c}}$ = creep curvature correction at t^{*} to allow for steel and tendon in section
ε = strain
$\varepsilon_{\mathrm{a.csh}}$ = concrete strain due to creep and shrinkage in the top fibre of a section
ε_{c} = strain in concrete
$\varepsilon_{\mathrm{cc}}$ = creep strain in concrete
$\varepsilon^{*}_{\mathrm{cc}}$ = long-term value of creep strain $\varepsilon_{\mathrm{cc}}$
$\varepsilon^{*}_{\mathrm{cc.eq}}$ = long-term free creep strain in section at depth d_{eq}
$\varepsilon_{\mathrm{ce}}$ = elastic strain in concrete
$\varepsilon_{\mathrm{cea}}$ = elastic strain in the concrete, top fibre
$\varepsilon_{\mathrm{ceb}}$ = elastic strain in the concrete, bottom fibre
$\varepsilon_{\mathrm{csh}}$ = shrinkage strain in concrete
$\varepsilon_{\mathrm{c.csh}}$ = concrete strain due to creep and shrinkage
$\varepsilon_{\mathrm{ce}}$ = initial elastic strain in concrete due to an applied stress
= elastic strain due to effective prestress in the concrete at tendon level
$\varepsilon_{\mathrm{ci}}$ = elastic strain due to initial prestress in the concrete at tendon level
$\varepsilon_{\mathrm{cp}}$ = concrete strain at tendon level
$\varepsilon_{\mathrm{cs}}$ = AS 3600 terminology for design shrinkage strain in concrete
$\varepsilon^{*}_{\mathrm{cs}}$ = long-term shrinkage strain in concrete
$\varepsilon_{\mathrm{csd}}$ = drying shrinkage strain
$\varepsilon_{\mathrm{cse}}$ = autogenous shrinkage strain
$\varepsilon_{\mathrm{cu}}$ = maximum compressive strain in the concrete of a cross-section in flexure
ε_{o} = strain at the peak stress of concrete in compression
ε_{o}, ε_{c} = compressive strain in concrete
ε_{p} = strain in the prestressing tendon
$\varepsilon_{\mathrm{pe}}$ = strain in prestressing tendon due to effective prestress

ε_{pi} = initial strain in prestressing tendon immediately after transfer
ε_{sc} = strain in the compressive steel reinforcement
ε_{sh} = total shrinkage strain for calculating deflection in a slab
ε_{st} = strain in the tensile steel reinforcement
ε_{sy} = yield strain of steel reinforcement
ε_{u} = concrete extreme fibre compressive strain at M_u
ε^*_{cc} = final creep strain in the concrete
ε^*_{cs} = final shrinkage strain in the concrete
ε_{cs} = design shrinkage strain according to AS 3600
η = friction loss factor
θ = an angle
= slope of prestressing cable
κ = curvature
$\kappa_L, \kappa_M, \kappa_R$
= curvatures at the left, middle and right ends of the beam
κ_o = initial elastic curvature at time t_o
$\kappa_c(t)$ = creep curvature at time t
κ^*_c = total long-term creep curvature at time t^*
κ^*_{co} = long-term free creep curvature at time t^*
κ^{**}_c = $\kappa^*_{co} + \Delta\kappa^*_c$ improved estimate of long-term creep curvature
κ_{sh} = shrinkage curvature
κ^*_{sh} = long-term shrinkage curvature
μ = coefficient of friction
ρ = the density of concrete
Σ = algebraic summation
σ = stress normal to a section
σ_1 = principal tensile stress
σ_{ca} = concrete top fibre stress
σ_{cb} = concrete bottom fibre stress
σ_{cbp} = concrete bottom fibre stress due to prestress
σ_{cc} = constant sustained concrete compressive stress
σ_{ce} = elastic stress due to effective prestress in the concrete at tendon level

σ_{ci} = elastic stress due to initial prestress in the concrete at tendon level

σ_{cs} = concrete (tensile) stress in a cross-section caused by shrinkage

σ_s, σ_{st} = tensile stress in steel reinforcement

σ_{sc} = compressive stress in steel reinforcement

σ_{scr} = stress in the tensile reinforcement obtained from a cracked section analysis

σ_{pa} = concrete top fibre stress due to prestress

= stress in prestressing cable at distance L_{pa} from the jack

σ_{pb} = concrete bottom fibre stress due to prestress

σ_{pj} = stress in prestressing cable at the jack

σ_{pu} = stress in prestressing steel at M_u

τ = shear stress

ϕ = capacity reduction factor

ϕ_{sys} = capacity reduction factor for a structural system

φ_{cc} = creep coefficient as used in AS 3600

$\varphi_{cc.b}$ = basic value of φ_{cc}, for $t_o = 28$ days, as used in AS 3600

$\varphi(t, t_o)$ = creep function; value at time t due to sustained stress applied at t_o

φ^*_{cc} = long-term value of $\varphi(t, t_o)$ at $t = t^*$, for stress applied at t_o

χ = aging coefficient for use with E_R

ψ_c = combination factor for imposed actions (live loads)

ψ_s = short-term live load factor used to determine the serviceability load

ψ_l = long-term live load factor used to determine the serviceability load

CHAPTER 1

Introduction

The basic ideas of prestressed concrete are introduced in this chapter. We explain what prestressing is and the advantages and disadvantages of prestressing concrete members. Methods of post-tensioning and pretensioning are explained. The chapter includes a short historical note on the development of prestressed concrete, from its beginnings at the end of the 19th Century.

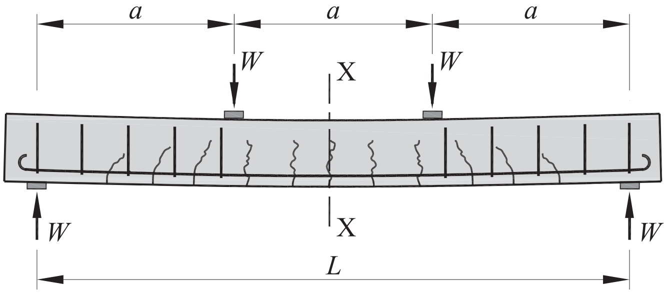
1.1 Prestressed concrete

1.1.1 Plain concrete and reinforced concrete

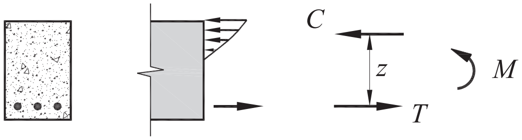
When external load is applied to structural members such as beams and slabs, large regions in the member are subjected to tensile stress. Tensile stresses may also be induced in structural members by load-independent effects such as temperature gradients and imposed deformations due to foundation movement. Because of its very low tensile strength, plain concrete cannot be used to construct such members where significant tension is present.

The compressive strength of concrete is reasonably good and if small amounts of reinforcing steel are placed in strategic locations in the concrete to carry the internal tensile forces that develop, an effective load-carrying mechanism is created. The resulting composite material is **reinforced concrete**. The great advantage of concrete as a building material is that it is very cheap, and even when reinforcing steel is added in small quantities, the cost advantage is still sufficient to ensure that reinforced concrete is presently the most widely used structural material, both in Australia and world-wide.

The flexural behaviour of reinforced concrete is illustrated in Figure 1.1, where the beam is supported on a simple span of length *L* and has two equal point loads *W* acting at the third points of the span which are located at distance $a = L/3$ from the supports.



(a) Reinforced concrete beam under service load



(b) Stresses and internal forces at section X-X

Figure 1.1 Reinforced concrete beam with external loads

Tensile stresses develop in the lower fibres of the beam as the first increments of load are applied. With increasing load, cracks soon appear throughout the mid-span region, where the moment is largest. At any cracked cross-section X-X between the load points (Figure 1.1(b)), the internal moment *M* is resisted by a tensile force *T* in the steel, located in the lower cracked region of the concrete, and an equal compressive force *C* in the intact compressive concrete above the crack. The steel is effective in carrying the tensile force in a cracked section, but it does not prevent or delay cracking of the tensile concrete. That is not its function. With increasing load the cracked region extends outwards towards each support, and the beam deflection increases. Under full service load, a well-developed pattern of fine cracks is present in the lower fibres of the beam.

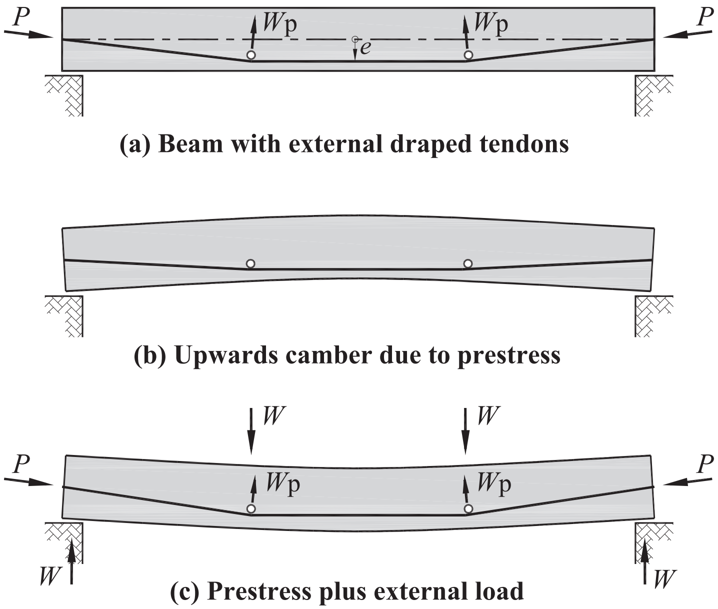
With overload, the existing cracks widen and the cracked region extends even further outwards. At high overload, the steel reinforcement in the mid-span region yields. The cracks then widen even more and the deflection increases rapidly with only a very slight further increase in load. Eventually the ultimate moment M_u of the sections is reached in the mid-span region and the beam fails in flexure at its load capacity W_max. In the design of reinforced concrete flexural members, the aim is to achieve good service load behaviour, in particular with narrow crack widths and small deflections, and adequate strength to prevent premature failure.

1.1.2 Prestressed concrete

Prestressing is another way of circumventing the poor tensile strength of plain concrete: a system of permanent compressive stresses is introduced into the regions of a concrete member where tensile stresses will subsequently develop when the external service loads act. This pre-compression delays tensile cracking, and may even prevent it altogether at service loads. The downward deflection due to external load is also reduced. Prestressing is, thus, an effective way of improving the service-load behaviour of a reinforced concrete member. Compressive prestress in the concrete cross-sections is usually achieved by the use of highly stressed, high-strength tensile steel or fibre reinforced plastic (FRP) tendons that run through the length of the member. The tendons are permanently anchored to the concrete at each beam end. At each internal cross-section, the tensile force in the tendon produces equilibrating compressive stresses in the concrete.

To illustrate the use of prestressing, we return to the reinforced concrete beam in Figure 1.1 and we consider the effect of stressing two draped external steel prestressing tendons, placed on the side faces as shown in Figure 1.2(a). At the ends, the tendons are anchored to the concrete at the section mid-depth. In-span they are draped around cast-in-place pins located in the lower fibres of the concrete, directly under the load points. In the mid-span region the eccentricity of the tendons, relative to the section centroid, is e and the total tensile force in the two tendons is P. The prestressed tendons apply forces to the concrete at the pins and at the end anchors. This is shown in Figure 1.2(a).

The upward force at each pin, W_p, is slightly inclined from the vertical, and the force P at each end is slightly inclined from the horizontal. These forces are self-equilibrating. The overall effect of the prestress is to create an upwards camber in the beam, as in Figure 1.2(b). When the external loads W are applied there is a downwards deflection (Figure 1.2(c)), but it is reduced by the prior prestress.



(a) Beam with external draped tendons

(b) Upwards camber due to prestress

(c) Prestress plus external load

Figure 1.2 Prestressed concrete beam, draped tendons

To investigate the stresses in the concrete due to the prestress, we consider the free body to the left of section X-X in the central region, as shown in Figure 1.3. At X-X the tensile prestressing force P in the tendons is horizontal, and at eccentricity e. This induces compressive stresses in the concrete, which have a resultant force $C = P$ that also must act at eccentricity e. The eccentric force C is statically equivalent to a compressive force C acting at the centroid of the section, plus a negative moment $M_p = Ce$, as in Figure 1.4(a). The concrete stresses are thus the sum of a uniformly distributed compressive stress C/A_c and the bending stresses due to M_p. At the bottom and top fibres, the stresses are:

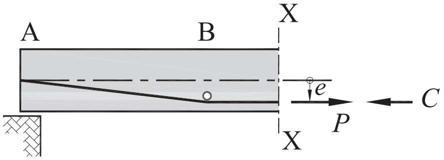


Figure 1.3 Equilibrating forces at section X-X

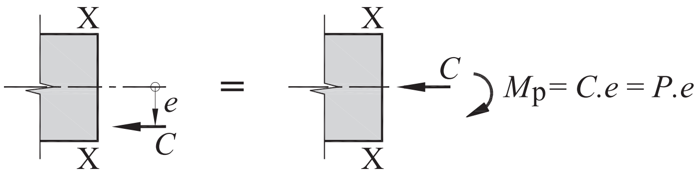
$$\sigma_{pb} = P\left[\frac{1}{A_c} + \frac{e}{Z}\right] \tag{1.1}$$

$$\sigma_{pa} = P\left[\frac{1}{A_c} - \frac{e}{Z}\right] \tag{1.2}$$

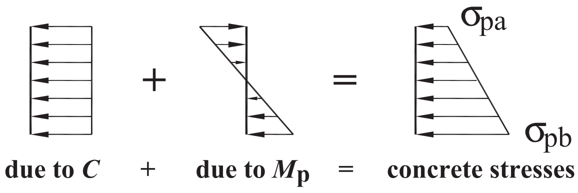
Here A_c is the area of the concrete cross-section and Z is its section modulus, which for a rectangular section is $bD^2/6$, where D is the section depth and b is the section width. The stress distribution in the section is shown in Figure 1.4(b). While the upper fibre stress σ_{pa} is shown as compressive, it will be tensile if the eccentricity e is sufficiently large, whereas the bottom fibre stress σ_{pb} is always compressive. The effect of any reinforcing steel in an uncracked section is very small and is ignored in Equations 1.1 and 1.2.

The external loads W induce a positive moment $M = Wa$ in the central region, with compressive stress in the upper fibres and tensile stress in the lower fibres. However, compressive stress is already present in the lower fibres of the section due to prestress. The resultant stresses at section X-X due to prestress plus external load are as shown in Figure 1.4(c).

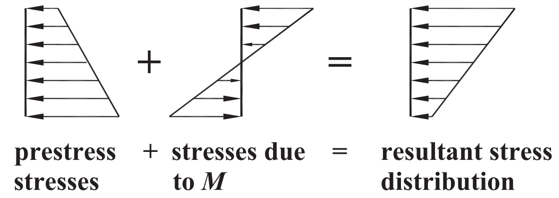
Whether or not the resultant stresses remain compressive in the bottom fibres, as shown in Figure 1.4(c), depends on the magnitudes of the prestressing force P, the eccentricity e and the load-induced moment, M. In any case, cracking will be delayed, or possibly even prevented, by the prestress. Also, the initial upwards deflection due to the prestress (Figure 1.2(b)) reduces, and may eliminate completely, the downwards deflection due to the external load W.



(a) Stress resultants at X-X due to prestress



(b) Stresses due to prestress

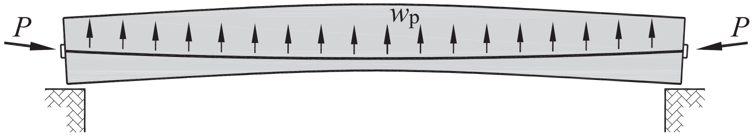


(c) Stresses due to prestress and external load

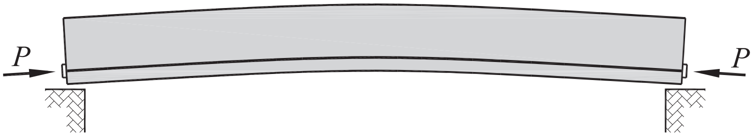
Figure 1.4 Concrete stresses at section X-X due to prestress and external load

In this example the prestressing tendons have been “draped”. The downward eccentricity in the middle region produces a negative moment $M_p = Pe$, which opposes the moment M due to the external loads. In the outer regions the eccentricity e reduces progressively to zero. The bottom fibre compressive stress due to prestress, σ_{pb}, reduces to the value P/A_c at the end of the beam, while the tensile stress due to external loading reduces to zero.

Various other tendon arrangements, different to the one shown in Figure 1.2, can be used to reduce cracking and deflection. For example, Figure 1.5(a) shows a curved tendon located in a parabolically shaped duct that has been cast in the concrete. The tendon is tensioned against the ends of the hardened concrete and then anchored permanently. As we shall see shortly, this form of construction is known as **post-tensioning**. The force in the tendon is *P* and the maximum eccentricity at mid-span, as before, is *e*. The upwards deflection due to the prestress is shown in the Figure.



(a) Prestressing with a parabolic tendon



(b) Prestressing with a straight eccentric tendon

Figure 1.5 Parabolic and straight tendons

The self-equilibrating force system exerted on the concrete by the curved prestressing tendon is shown in Figure 1.5(a). It consists of slightly inclined end forces *P* at the anchorage points, and an upward distributed force w_p acting along the beam in a direction perpendicular to the curved cable. If the cable curvature is constant (which is the case if the shape is parabolic) then it can be shown that w_p is a uniformly distributed force.

At mid-span the tendon force *P* is horizontal. It is equilibrated by a horizontal compressive force *C* in the concrete. The distribution of compressive concrete stresses here is therefore the same as that shown already in Figure 1.4(b). Although the eccentricity of the tendon decreases towards the supports, the prestressed tendon induces compressive stresses in the lower fibres at each section so that cracking is delayed or prevented when the load is applied. The total downwards deflection is also reduced.

In Figure 1.5(b) yet another tendon shape is shown. This time the tendon is straight, with the eccentricity *e* constant along the full length of the beam.

This profile is typical when **pre-tensioning** is employed. The straight tendon induces the same stress distribution in every section along the beam, and this is as shown in Figure 1.4(b). There is a uniform compression of P/A_c and bending stresses due to the moment $M_p = P\,e$. The overall effect is again to induce compressive stresses in the lower fibres and hence to delay or prevent cracking. The initial upwards camber acts to reduce or prevent deflection under external service load. However, the negative prestressing moment is now constant along the beam, and so becomes greater than the positive applied moment in the outer regions near the beam ends. In the design of such beams, care must be taken to ensure that the negative moment due to prestress is not excessive in the end regions.

For the beam in Figure 1.5(b), the self-equilibrating forces due to the straight tendon consists simply of equal and opposite horizontal forces P applied with eccentricity e at each end of the beam. This is statically equivalent to axial end forces P, plus a pair of negative end moments $M_p = -Pe$ at each end.

Figures 1.2 to 1.5 show how various tendon shapes are used to induce an equilibrating system of forces that act on the concrete. These forces act at the ends of the tendon where it is anchored to the concrete, and at any point along the span where the tensioned tendon changes direction. In particular, a concentrated force W_p acts at a kink in the tendon (as in Figure 1.2) and a distributed transverse force w_p acts over any length of member in which the tendon is curved (as in Figure 1.5(a)).

The forces exerted by the prestressed tendon on the beam can be thought of as **equivalent loads**. For example, in Figure 1.2 the equivalent loads are the upwards acting point loads W_p and the inclined end forces P, while in Figure 1.5(a) the equivalent loads consist of a uniformly distributed equivalent load w_p and the inclined end forces P.

The concept of equivalent loads gives us a simple but very useful view of the effect of prestress on the behaviour of members, both statically determinate and statically indeterminate. It also provides a convenient method for evaluating the stresses that are produced in a beam by the prestress. Furthermore, a simple but extremely useful design technique, called **load balancing**, can be developed from the equivalent load concept. Ideas of equivalent loads and load balancing are discussed in detail in Chapter 4.

THE IDEA OF PRESTRESSING

The idea of prestressing has wider applicability than in the field of prestressed concrete. A simple form of prestressing has been used by coopers for centuries to construct wine barrels by forcing heated metal tension bands over wooden staves. The precompression induced when the bands cool prestresses the staves together and prevents leaking. A variation of this technique is used today in the construction of large circular prestressed concrete liquid-retaining tanks. The procedure involves winding prestressing tendons around precast vertical concrete 'staves'.

One of the first suggestions to introduce prestress into structural concrete was made by P H Jackson in 1886 in San Francisco. A patent taken out in Berlin in 1888 by Doehring anticipated the idea of the production method which uses a pretensioning bed. Various proposals and tests followed, but this early developmental work was unsuccessful because mild steel reinforcing bars were used as the prestressing medium.

In the 1920s, R H Dill in the USA recognised that high strength wire could be used to produce a satisfactory prestressed member. However, the first successful practical designs in prestressed concrete were carried out in Europe by Eugene Freyssinet in the 1930s, when the time-dependent creep and shrinkage behaviour of concrete came to be better understood. In the United States, prestressed concrete was first used in the construction of circular water tanks. In the late 1930s the Preload Corporation developed the technique of winding wires around cast-in-place circular concrete walls.

Shortly after World War 2, Freyssinet designed a number of successful and highly acclaimed bridges in France, which led to wide acceptance of prestressed concrete. An upsurge in interest in prestressed concrete at that time can be attributed in part to the scarcity and high cost of steel and other structural materials in those post-war years. In the United States, prestressed concrete was first used in bridge construction in the late 1940s. Interesting historical information on the development of prestressed concrete, on personalities involved in its early development, and on the range of structures previously constructed in prestressed concrete, is to be found in the T Y Lin Symposium on Prestressed Concrete, reported in the Prestressed Concrete Journal (Lin, 1976).