

MATHEMATICS OF BUSINESS AND FINANCE

FOURTH EDITION



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DAISLEY • KUGATHASAN • HUYSMANS



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FOURTH EDITION

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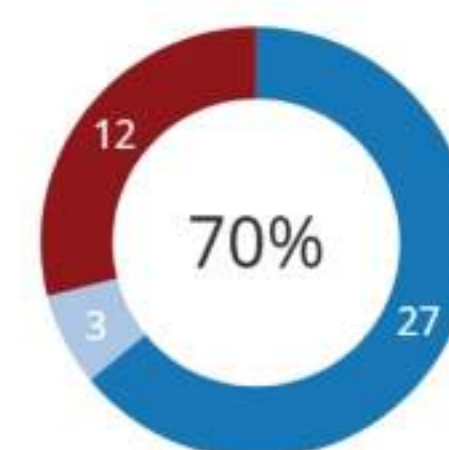
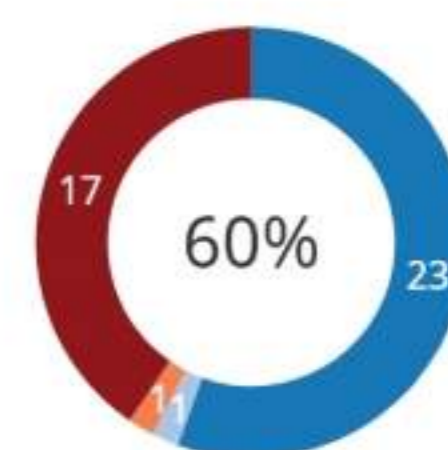
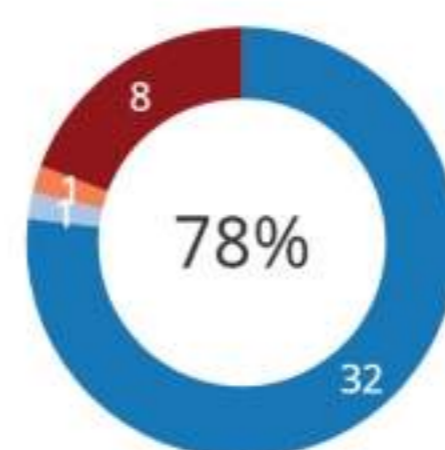
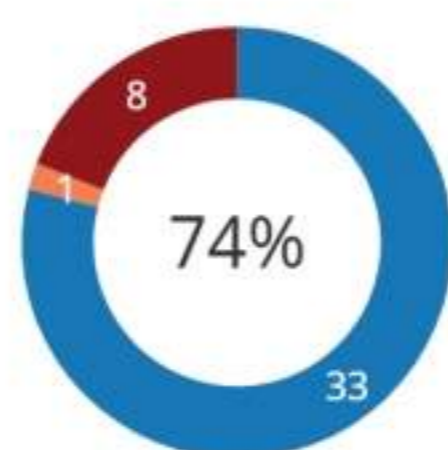
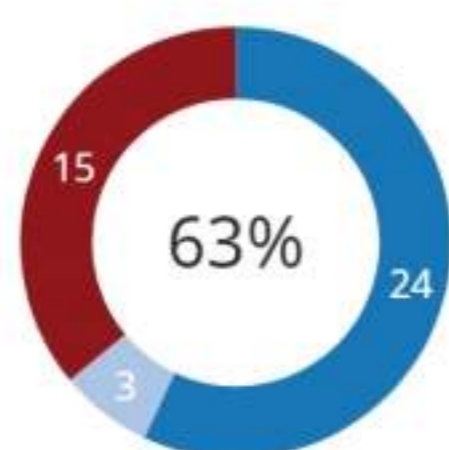
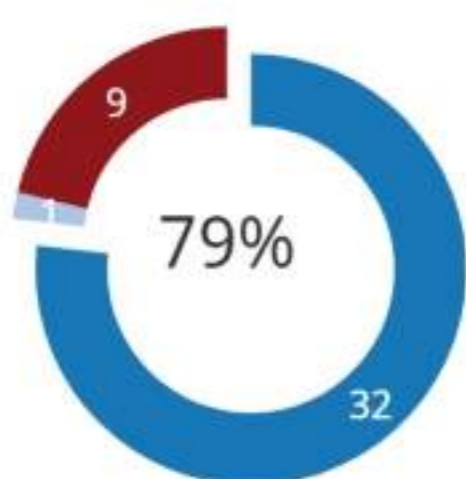
Preface

The *Mathematics of Business and Finance* resource was designed with a vision of creating a world where everyone enjoys mathematics. We set out to build a resource that will support students on their journeys to becoming future business leaders and help them develop a passion for mathematics along the way.

To optimize the learning experience, we have leveraged cutting-edge technologies to provide students with a highly immersive and engaging learning experience.

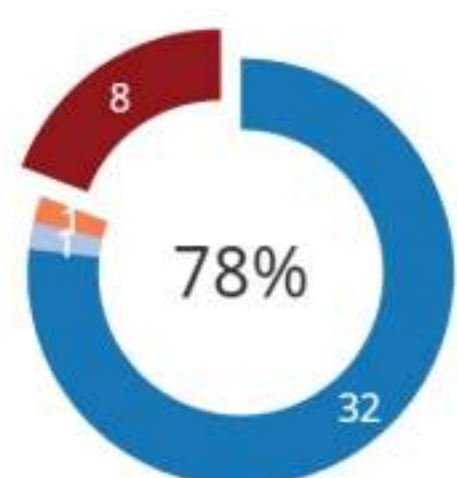
Knowing that every student learns differently, we created innovative online tools designed to help them master the concepts using diverse learning methods. Students have access to interactive online lessons which give them the opportunity to gain a conceptual understanding of topics before class. They are also provided with access to an online assignment system which tests them on all the core concepts covered throughout their course. Our aim in building these online tools is to help students overcome the negative perceptions and anxiety associated with mathematics.

- 80% or more
- 60% - 79%
- 50% - 59%
- 49% or less



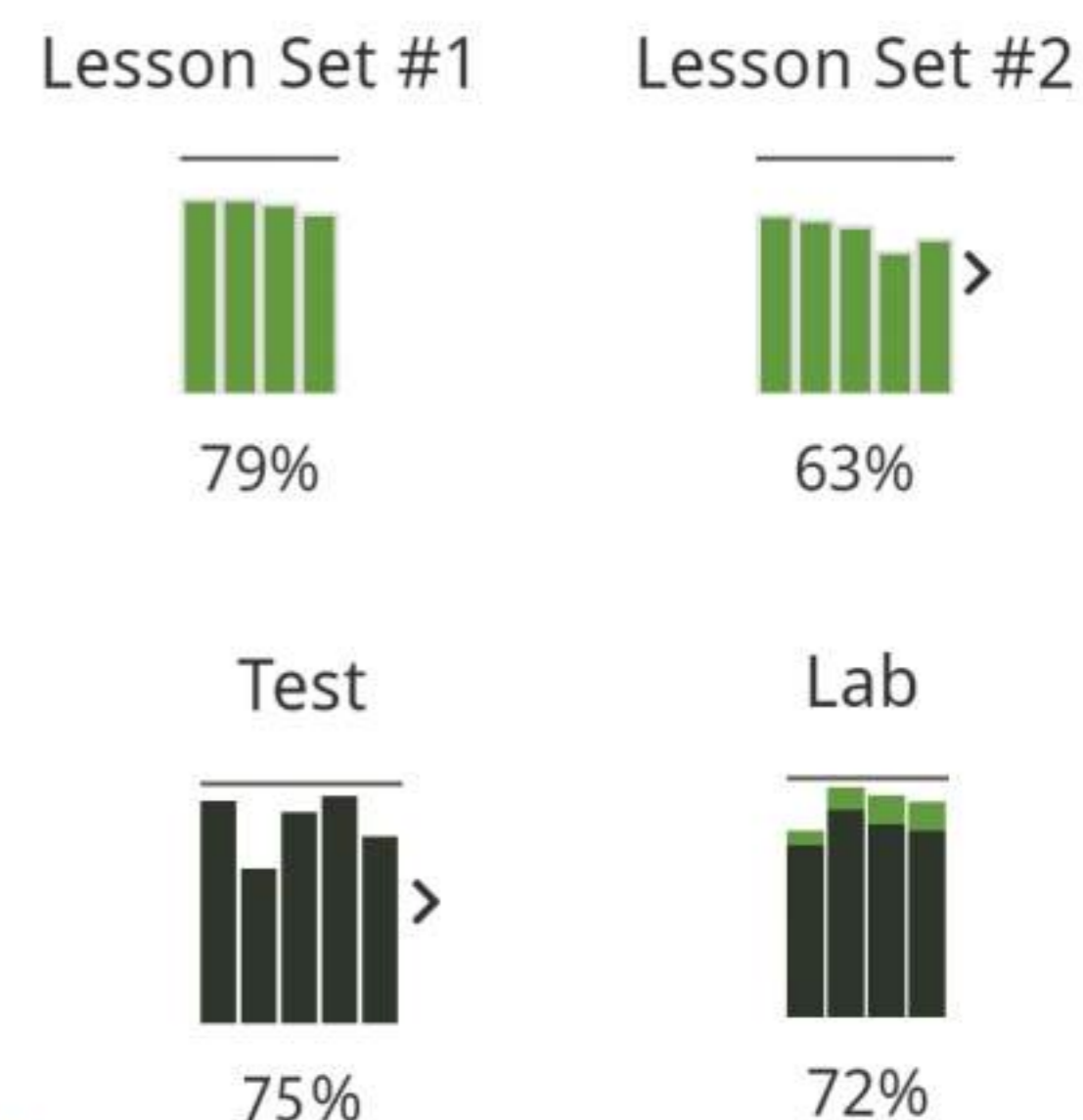
The concepts covered throughout these resources will support students dealing with practical scenarios such as repaying student loans, leasing a car, securing a mortgage, running a business, paying taxes, making investments, and much more. We have worked to build a resource that will equip our leaders of tomorrow with the core mathematical skills required to perform the day-to-day tasks demanded of them to be successful in their field of choice.

Our slogan is “Learn Math. Live Smart.”, and we truly believe that this blended resource will help students achieve a better understanding of the concepts presented and ultimately lead to a better overall quality of life. As students embark on the journey of learning this important subject, we hope that the [Mathematics of Business and Finance](#) resources will provide great value in helping them along the way.



Blended learning, commonly known as hybrid learning, is a method of combining different learning environments, such as face-to-face, web-based, and mobile learning, to enhance the learning experience for all types of learners (visual, auditory, tactile, sociological, etc.).

Vretta is a world-leader in delivering blended learning solutions that are transforming the way students engage with mathematics. We have received international recognition from prestigious awarding organizations like the Brandon Hall Group, e-Assessment Association, EdTech Digest, and the International Education Games Competition for increasing student success using innovative and engaging technology approaches for math education.



Resources


Textbook

Language The language used in this textbook is simple and straight-forward, while maintaining the levels of sophistication required to thoroughly prepare students for the next stage in their academic and professional careers.

Pedagogies and Learning Methods


Numerous pedagogies and learning methods that have been developed and proven over 30 years are incorporated into the textbook. These pedagogies simplify critical mathematical concepts and have successfully improved the retention of those concepts. The different learning methods to solve problems have also proven to cater to the varied learning styles of all students.

Calculator This textbook uses the pre-programmed Texas Instruments BA II Plus financial calculator to solve problems in conjunction with algebraic methods, wherever required. The calculator instructions are designed using the actual images of the calculator keys, making it very intuitive for students to follow the sequence of operations.

Spreadsheet Each of the finance chapters contains instructions on how to solve the in-text examples using spreadsheet software, wherever applicable. The spreadsheet instructions are designed to show students both the exact inputs they enter into each cell, as well as the final spreadsheet output. 

Exercises This textbook has over 2000 exercises, review exercises, self-test exercises, and comprehensive cases, as well as over 300 solved examples. The problems are designed to test students on real-world, practical applications and are presented in the order of the topics covered within the chapter sections, with the most difficult problems being indicated by a dot (•). The problems are categorized into pairs of similar questions to provide instructors with an opportunity to solve the even numbered exercises in class and assign the odd-numbered exercises as homework. New to the Fourth Edition, the exercises are grouped based on the content topics covered within the sections, to allow instructors and students to focus on certain sections, as required.

Financial Designation Problems

The textbook contains problems that are used to prepare students for the first level of the Chartered Financial Analyst (CFA) professional designation exam. This will provide students with exposure to the financial career choices that will be available to them in the future. These problems are identified by a 'CFA Prep.' icon. 

Solution Manual All problems in the end-of-section exercises, review exercises, self-test exercises, and cases have been solved using detailed step-by-step methods, including the financial calculator and spreadsheet methods, as demonstrated in the solved examples. The solution manual is available online.

Slide Decks

The animated slide decks (PowerPoint presentations) are available for instructors to use in class. They contain both the algebraic and calculator methods of solving problems. The slide decks are designed to teach every concept in the textbook. Steps in teaching concepts appear one by one through the creative use of animations to provide students with an inclusive learning experience. Instructors have the option of downloading and making modifications to any of the slide decks. Additionally, the decks are designed to be used with student response systems (clickers) to gauge the overall class comprehension of concepts.

Lessons

The interactive online lessons provide students with a unique *assessment for learning experience*. Through a step-by-step, mastery approach, students learn core concepts in mathematics. The lessons contain numerous pedagogies that break-down concepts into micro-steps which help students to visualize, conceptualize, and enjoy the learning process.

The lessons are modular in nature and can be customized to a large degree based on the learning plan of each instructor or the course. Students will be required to unlock



(master) key concepts, through built-in feedback mechanisms, before they proceed to higher level concepts. This approach has not only proven to help students to review concepts before tests and exams, but also to prepare them prior to attending class.

The system automatically records students' progress and performance. Once students complete a lesson, it unlocks, allowing students to navigate back and forth through the lesson. Instructors have administrative access which allows them to navigate through the online lessons without any restrictions.

Test Bank

The online assignment system contains a rich and comprehensive test bank of real-world problems that are algorithmically generated and provide students with dynamic feedback on their responses. The labs and assignments can be customized based on course requirements. Some of the customizable features include previewing and selecting questions, setting the number of questions, setting and modifying start and due dates, opening, closing and re-opening labs, creating new labs and quizzes, and determining the weighting and number of attempts for each question. In addition to providing students with the ability for unlimited practice on each question, the assessments contain unique features such as the provision for students to "Check My Answer", "Ask My Professor", and also receive dynamic feedback based on the entered answer.

Assignments and Tests

Assignments and tests can be customized by the instructors or administrators to create a secure and accessible environment for administering in-class or remote assessments. The online assessment settings have been developed to provide a high level of security. Features such as setting a password, restricting feedback and marks, and questions shuffling, can be activated to minimize student collaboration. Additionally, accessibility features, such as extending time and dates for individual students or having a printed version of the exam, are available to ensure that students with accommodations can be successfully tested with minimal barriers.

There are also many other available features, such as setting and modifying grades, multiple attempts, hiding previously correct questions, or question by question submission, which can be used by instructors or administrators to customize their tests and exams to meet their educational goals.

Administrative Tools

Performance Dashboard for Instructors

The instructor dashboard provides a real-time visual understanding of individual student and overall class performance as they progress through the lessons and assessments.

While viewing lesson performance, instructors can see the average completion percent per chapter, including a lesson-by-lesson percent completed visualization for the entire class.

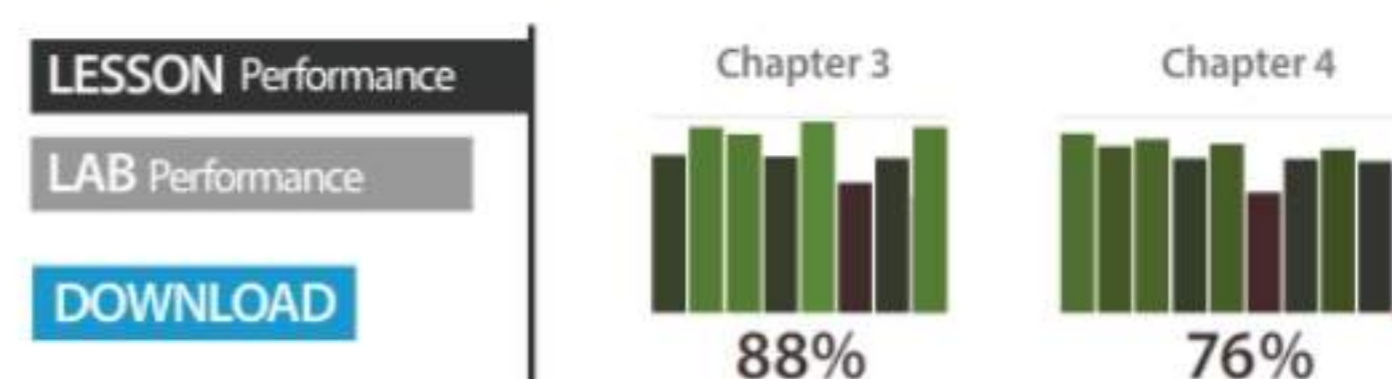
While viewing assessment performance, the dashboard provides instructors with the average percent mark on each assignment, test, or exam. They can also see the highest mark and average mark secured on every question. This helps to identify the specific concept that an individual student or the entire class is having difficulty understanding. Instructors can also view the individual submission from each student, question by question. In this view, they can review and override marks as needed.

Instructors can also download or export individual grades for lessons and assessments to a spreadsheet or synchronize the grades to the institution's learning management system.

Performance Dashboard for Students

Students have access to dashboards which provide them with a visualization of their progress on their lessons and assessments. From their dashboard they can see their chapter completion marks, including a lesson-by-lesson percent completed visualization, as well as their marks on their assignments, tests, or exams.

These visualizations allow students to see their performance over time, which helps them to determine their areas of strength and concepts that require more work to master.



Updates in the Fourth Edition

The content of the Fourth Edition textbook along with its 300+ solved examples, 2000+ exercises, and cases have been carefully reviewed by professors at various colleges in Canada. In addition to the updates to the textbook, accompanying online resources have also been updated to reflect the changes.



Spreadsheet Solutions

Full Spreadsheet Solutions provided for over **130 Examples** in Chapters 8-14.

Updates to Exercise Questions

- Exercise questions, at the end of each section and Review Exercises at the end of each chapter, have been grouped and categorized based on the content topics covered within the sections.
- Over **200 New Exercise Questions** added throughout the textbook.

New Cases

Added **1 New Case Study** in Chapter 13

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New Examples

Examples revised, updated, and expanded throughout the textbook, with **31 New Solved Examples** added

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Section 1.2:	Modified examples and added tree diagrams for clarity
Section 1.3:	Included steps for each operation with fractions
Section 1.3:	Added definitions of key terms
Chapter 3:	Added Algebraic Method solutions to all portion, rate, base examples
Section 2.2:	Clarified steps for solving word problems
Section 3.1:	Reorganized order of converting between decimals, fractions, and percents
Section 4.1:	Sharing Quantities Using Ratios
Section 4.3:	Clarified definitions of buy and sell rate
Chapter 9:	Combined equivalent and effective interest rate in one section

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MATHEMATICS OF BUSINESS AND FINANCE

FOURTH EDITION

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Chapter

1

REVIEW OF BASIC ARITHMETIC

LEARNING OBJECTIVES

- Identify place values of numbers and apply to round whole numbers, decimal numbers, and percents to the required place values.
- Determine least common multiples or denominators (LCM or LCD) and greatest common factors (GCF).
- Identify types of fractions and perform computations using fractions.
- Perform basic arithmetic operations by following the order of operations with integers, decimals, and fractions.
- Calculate simple arithmetic averages, weighted averages, and geometric averages for a set of values.

CHAPTER OUTLINE

- 1.1 Place Value of Numbers and Rounding Numbers
- 1.2 Factors and Multiples
- 1.3 Fractions
- 1.4 Order of Operations (BEDMAS)
- 1.5 Averages

Introduction

Arithmetic

is the study of numbers and includes calculations, such as addition, subtraction, multiplication, and division, that may be performed between them.

Arithmetic is the most elementary branch of mathematics. It is the study of numbers and includes calculations such as addition, subtraction, multiplication, division, etc. We use arithmetic in everyday tasks such as counting, buying, selling, estimating expenses, and checking bank balances. Arithmetic also forms the basis for all advanced technology, science, engineering, and business studies.

Throughout this textbook, we will be deriving multiple formulas related to the mathematics of business and finance. Before we can understand how these formulas work and how to properly apply them, it is essential that we gain confidence in performing arithmetic operations in the right order, using whole numbers, decimal numbers, and fractions.

In this chapter, you will review the basic arithmetic skills that are necessary for these business and finance applications.

1.1 Place Value of Numbers and Rounding Numbers

Place Value of Whole Numbers

The position of each digit in a whole number determines the **place value** for the digit. Exhibit 1.1(a) illustrates the place value of the ten digits in the whole number 3,867,254,129. In this example, 4 is in the 'thousands' place value and represents 4,000, whereas 7 is in the 'millions' place value and represents 7,000,000.

We read and write numbers from left to right. A comma (or alternatively, a space) separates every three digits into groups, starting from the place value for 'ones', thereby making it easier to read a whole number.

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	8	6	7	2	5	4	1	2	9

Exhibit 1.1(a): Place Value of a Ten-Digit Whole Number

Table 1.1(a)

Place Value Chart of Whole Numbers

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1,000,000,000	100,000,000	10,000,000	1,000,000	100,000	10,000	1,000	100	10	1
10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0

The place value of 'ones' is 1 ($= 10^0$) and each place has a value 10 times the place value to its right, as shown in Table 1.1(a) above. The use of exponential notation in representing each place value as powers of 10 is illustrated in Section 1.4.

The red, vertical lines denote the positions of the commas (or spaces) that separate the groups of three numbers, starting from the place value for 'ones'. When written in **standard form**, the ten-digit whole number in Exhibit 1.1(a) is written as 3,867,254,129.

3	8	6	7	2	5	4	1	2	9
---	---	---	---	---	---	---	---	---	---

This can be written in **expanded form** as follows:

$$\begin{aligned} & \blacksquare 3,000,000,000 + 800,000,000 + 60,000,000 + 7,000,000 + 200,000 \\ & \quad + 50,000 + 4,000 + 100 + 20 + 9 \end{aligned}$$

OR,

$$\begin{aligned} & \blacksquare 3 \text{ billion} + 800 \text{ million} + 60 \text{ million} + 7 \text{ million} + 200 \text{ thousand} \\ & \quad + 50 \text{ thousand} + 4 \text{ thousand} + 1 \text{ hundred} + 2 \text{ tens} + 9 \text{ ones} \end{aligned}$$

Reading and Writing Whole Numbers

To make it easier to read and write numbers, any number larger than three digits is separated into smaller groups of three digits, starting from the last digit of the number. Each group of these three digits has a name. For example, the first group of three digits on the right is the **units** group.

Trillions			Billions			Millions			Thousands			Units		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Follow these steps to write large numbers in **word form**:

Step 1: Start from the group furthest to the left and write the number formed by the digits in that group, followed by the name of the group.

Step 2: Moving to the next group (to the right), write the numbers formed by this next group, followed by its name. Continue to do this for each of the groups.

Step 3: For the last group (i.e. the group furthest to the right), write the numbers formed by the group; however, for this group, do not write the name of it.

Note: When all three digits in a group are zero, that group is neither read nor written.

Additionally, commas and hyphens are used when expressing numbers in word form.

- Commas (,) are used between the groups to separate them.
- Hyphens (-) are used to express the two-digit numbers in each group; i.e. 21 to 29, 31 to 39, 41 to 49, ... 91 to 99.

For example, we write 700,629 in word form as: Seven hundred thousand, six hundred twenty-nine.

Place Value of Decimal Numbers

The position of each digit in a decimal number determines the place value of the digit. Exhibit 1.1(b) illustrates the place value of the five-digit decimal number: 0.35796.

The place value of each digit is found by decreasing powers of 10, as shown in Table 1.1(b) below:

	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
0	.	3	5	7	9	6

Exhibit 1.1(b): Place Value of a Five-Digit Decimal Number

The word 'and' does not appear in the word form of whole numbers.

Table 1.1(b)

Place Value Chart of Decimal Numbers

Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
0.1	0.01	0.001	0.0001	0.00001
$10^{-1} = \frac{1}{10}$	$10^{-2} = \frac{1}{100}$	$10^{-3} = \frac{1}{1,000}$	$10^{-4} = \frac{1}{10,000}$	$10^{-5} = \frac{1}{100,000}$

For example, the five-digit decimal number in Exhibit 1.1(b) is written as 0.35796 in **standard form**.

0.	3	5	7	9	6
----	---	---	---	---	---

This can be written in **expanded form** as follows:

3 tenths + 5 hundredths + 7 thousandths + 9 ten-thousandths + 6 hundred-thousandths

OR,

$0.3 + 0.05 + 0.007 + 0.0009 + 0.00006$

OR,

$$\frac{3}{10} + \frac{5}{100} + \frac{7}{1000} + \frac{9}{10,000} + \frac{6}{100,000} = \frac{35,796}{100,000}$$

Reading and Writing Decimal Numbers

Follow these steps to read and write decimal numbers in **word form**:

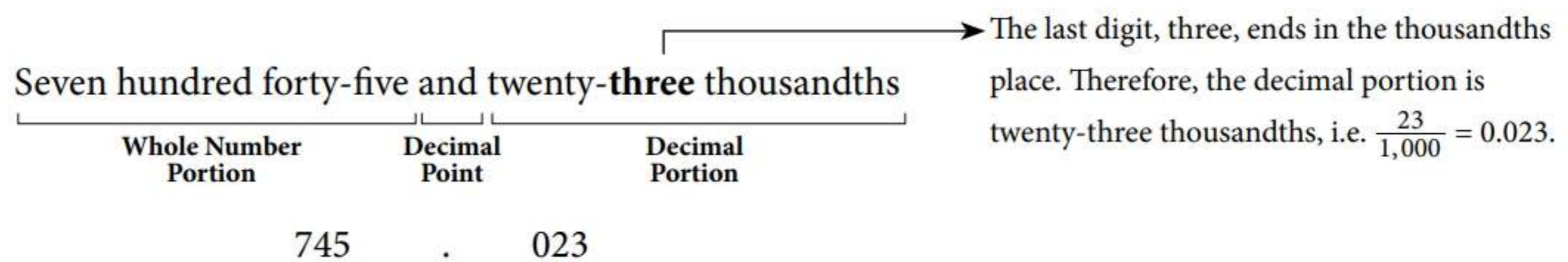
Step 1: Read or write the numbers to the left of a decimal point as a whole number.

Step 2: Read or write the decimal point as 'and'.

Step 3: Read or write the number to the right of the decimal point also as a whole number, but followed by the name of the place value occupied by the digit on the far right.

■ Hyphens (-) are used to express the place value portion of a decimal number such as ten-thousandths, hundred-thousandths, ten-millionths, hundred-millionths, etc.

For example, we write 745.023 in word form as:



There are other ways of reading and writing decimal numbers as noted below.

(i) Use the word 'point' to indicate the decimal point, and, thereafter, read or write each digit individually.

For example, we read or write 745.023 as: Seven hundred forty-five point zero, two, three.

(ii) Ignore the decimal point of the decimal number and read or write the number as a whole number and include the place value occupied by the digit on the far right of the decimal number.

For example, we read or write 745.023 as: Seven hundred forty-five thousand, twenty-three thousandths; i.e. $\frac{745,023}{1,000}$.

Note: The above two representations are not used in the examples and exercise questions within this chapter.

The word 'and' is used to represent the decimal point (.).

Example 1.1(a)**Identifying the Place Value of a Digit and the Amount It Represents**

What is the place value of digit 5 in each of the following numbers and what amount does it represent?

- (i) 675,342 (ii) 35,721,890 (iii) 243.4759 (iv) 64.057 (v) 231.547

Solution

	(i) 675,342	(ii) 35,721,890	(iii) 243.4759	(iv) 64.057	(v) 231.547
Place Value of Digit 5	↑ Thousands	↑ Millions	↑ Thousandths	↑ Hundredths	↑ Tenths
Amount it Represents	5,000	5,000,000	0.005	0.05	0.5

Example 1.1(b)**Identifying the Digit of a Number Given its Place Value**

In the number 320,948.751, identify the digit that occupies the following place values:

- (i) Hundred thousands (ii) Ten thousands (iii) Thousands (iv) Hundredths (v) Tenths

Solution

- (i) 3; (320,948.751) (ii) 2; (320,948.751) (iii) 0; (320,948.751) (iv) 5; (320,948.751) (v) 7; (320,948.751)

Example 1.1(c)**Writing Numbers in Expanded and Word Forms Given Their Standard Forms**

Write the following numbers in expanded and word forms:

- (i) 43,583,621 (ii) 8,213,505,235 (iii) 478.25 (iv) 22.006 (v) 213.5078

Solution

- (i) Standard form: 43,583,621
Expanded form: $40,000,000 + 3,000,000 + 500,000 + 80,000 + 3,000 + 600 + 20 + 1$
Word form: Forty-three million, five hundred eighty-three thousand, six hundred twenty-one
- (ii) Standard form: 8,213,505,235
Expanded form: $8,000,000,000 + 200,000,000 + 10,000,000 + 3,000,000 + 500,000 + 5,000 + 200 + 30 + 5$
Word form: Eight billion, two hundred thirteen million, five hundred five thousand, two hundred thirty-five
- (iii) Standard form: 478.25
Expanded form: $400 + 70 + 8 + 0.2 + 0.05$
Word form: Four hundred seventy-eight and twenty-five hundredths
- (iv) Standard form: 22.006
Expanded form: $20 + 2 + 0.006$
Word form: Twenty-two and six thousandths
- (v) Standard form: 213.5078
Expanded form: $200 + 10 + 3 + 0.5 + 0.007 + 0.0008$
Word form: Two hundred thirteen and five thousand, seventy-eight ten-thousandths

Example 1.1(d)**Writing Numbers in Standard Form Given Their Word Form**

Write the following in standard form:

- (i) Thirty-five thousand, eight hundred twenty-five
(ii) Three million, three hundred forty-two thousand, six hundred seventeen
(iii) Half of a million
(iv) Three-quarters of a billion

Solution

(i) 35,825

(ii) 3,342,617

(iii) Half of a million is $\frac{1}{2} \times 1,000,000 = 500,000$

(iv) Three-quarters of a billion is $\frac{3}{4} \times 1,000,000,000 = 750,000,000$

Rounding Whole Numbers and Decimal Numbers

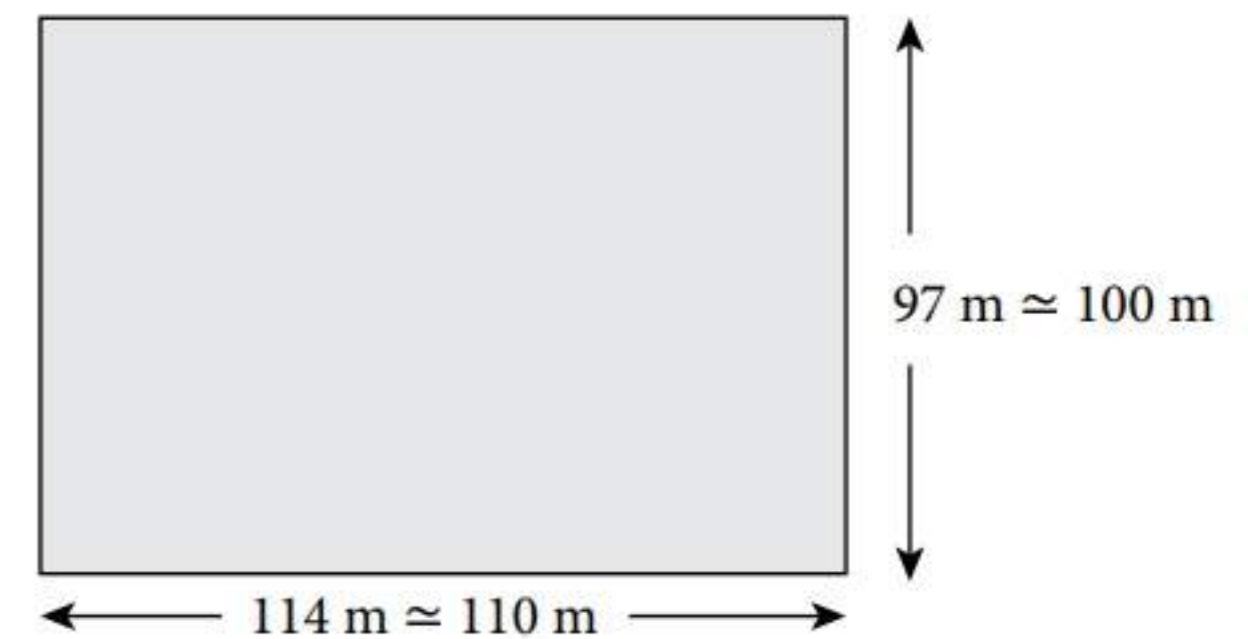
Rounding means making a number simpler to use while keeping its value close to the original value.

Rounding numbers makes them easier to work with and easier to remember. Rounding changes some of the digits in a number but keeps its value close to the original. It is used in reporting large quantities or values that change often, such as in population, income, expenses, etc.

For example, the population of Canada is approximately 37 million, or Dominic's car expense for this month is approximately \$900.

Rounding numbers also makes arithmetic operations faster and easier when it is not required to calculate the exact answer.

For example, if you are required to estimate the area of a rectangular plot of land that measures 114 m by 97 m, you would have to multiply 114×97 , which will result in $11,058 \text{ m}^2$. However, you can get a quick estimate by rounding the measurements to the nearest ten. Rounding 114 m to the nearest ten is 110 m. Rounding 97 m to the nearest ten is 100 m. This will result in an estimated area of $110 \text{ m} \times 100 \text{ m} = 11,000 \text{ m}^2$.



Rounding Whole Numbers to the Nearest Ten, Hundred, Thousand, etc.

Rounding whole numbers refers to changing the value of the whole number to the nearest ten, hundred, thousand, etc. It is also referred to as rounding whole numbers to multiples of 10, 100, 1000, etc.

For example,

- Rounding a whole number to the nearest ten is the same as rounding it to a multiple of 10.
- Rounding a whole number to the nearest hundred is the same as rounding it to a multiple of 100.
- Rounding an amount to the nearest \$10 refers to rounding the amount to a multiple of \$10.

Rounding Decimal Numbers to the Nearest Whole Number, Tenth, Hundredth, etc.

Rounding decimal numbers refers to changing the value of the decimal number to the nearest whole number, tenth, hundredth, thousandth, etc. It is also referred to as "rounding to a specific number of decimal places", indicating the number of decimal places that will be left when the rounding is complete.

For example,

- Rounding to the nearest whole number is the same as rounding without any decimals.
- Rounding to the nearest tenth is the same as rounding to one decimal place.
- Rounding to the nearest hundredth is the same as rounding to two decimal places.
- Rounding to the nearest cent refers to rounding the amount to the nearest hundredth or to two decimal places.

Rounding whole numbers and **decimal numbers** refers to rounding up or down to the desired place value.

If the place value for which you are rounding is followed by 5, 6, 7, 8, or 9, increase the value of the rounding digit by 1.

If the place value for which you are rounding is followed by 0, 1, 2, 3, or 4, do not change the value of the rounding digit.

Rules for Rounding Whole Numbers and Decimal Numbers

- Step 1:** Identify the digit to be rounded (this is the place value for which the rounding is required).
- Step 2:** If the digit to the immediate right of the required rounding digit is less than 5 (0, 1, 2, 3, 4), do not change the value of the rounding digit.
- If the digit to the immediate right of the required rounding digit is 5 or greater than 5 (5, 6, 7, 8, 9), increase the value of the rounding digit by one (round up by one number).
- Step 3:** For rounding whole numbers: after Step 2, change the value of all the digits that are to the right of the rounding digit to 0.
For rounding decimal numbers: after Step 2, drop all the digits that are to the right of the rounding digit.

Example 1.1(e)

Rounding Whole Numbers to Indicated Place Values

Round the following whole numbers to the indicated place value:

- (i) 18,568 to the nearest ten
- (ii) \$24,643 to the nearest \$100

Solution

- (i) *Rounding 18,568 to the nearest ten*
 - Identify the rounding digit in the tens place: 18,568.
 - The digit to the immediate right of the rounding digit is 8, which is greater than 5; therefore, increase the value of the rounding digit by one, from 6 to 7, and change the value of the digits that are to the right of the rounding digit to 0, which will result in 18,570.

Therefore, 18,568 rounded to the nearest ten (or multiple of 10) is 18,570.
- (ii) *Rounding \$24,643 to the nearest \$100*
 - Identify the rounding digit in the hundreds place: 24,643.
 - The digit to the immediate right of the rounding digit is 4, which is less than 5; therefore, do not change the value of the rounding digit, but change the value of the digits that are to the right of the rounding digit to 0, which will result in 24,600.

Therefore, \$24,643 rounded to the nearest \$100 (or multiple of \$100) is \$24,600.

Example 1.1(f)

Rounding Numbers (Visual Method Using a Number Line)

Round the following numbers to the indicated place value:

- (i) 627 to the nearest ten (multiples of 10)
- (ii) 16.5 to a whole number

Solution

We can visualize these numbers on a number line to determine the nearest number.

- (i) *Rounding 627 to the nearest ten (multiples of 10)*

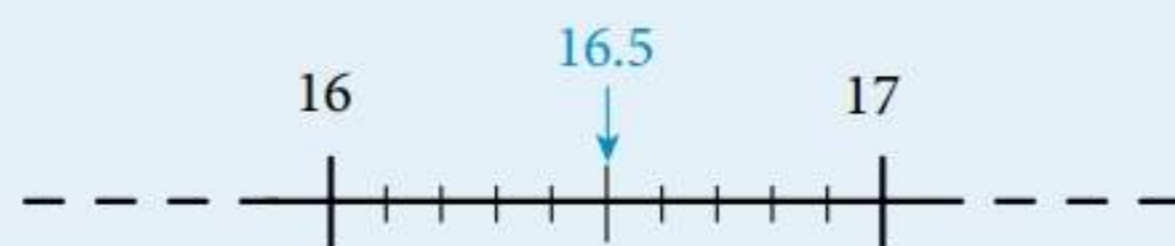


627 is closer to 630 than 620.

Therefore, 627 rounded to the nearest ten is 630.

Solution
continued

- (ii) *Rounding 16.5 to a whole number*



16.5 is at the mid-point of 16 and 17. By convention, if a number is exactly in the middle, we round it up.

Therefore, 16.5 rounded to a whole number is 17.

Example 1.1(g)

Rounding Decimal Numbers

Round the following decimal numbers to the indicated place value:

- (i) 268.143 to the nearest hundredth
- (ii) \$489.677 to the nearest cent
- (iii) \$39.9985 to the nearest cent

Solution

- (i) *Rounding 268.143 to the nearest hundredth*

- Identify the rounding digit in the hundredths place: 268.143.
- The digit to the immediate right of the rounding digit is less than 5; therefore, do not change the value of the rounding digit. Drop all the digits to the right of the rounding digit, which will result in 268.14.

Therefore, 268.143 rounded to the nearest hundredth (or to two decimal places) is 268.14.

- (ii) *Rounding \$489.677 to the nearest cent*

- Identify the rounding digit in the hundredths place: \$489.677.
- The digit to the immediate right of the rounding digit is greater than 5; therefore, increase the value of the rounding digit by one, from 7 to 8, and drop all the digits that are to the right of the rounding digit, which will result in \$489.68.

Therefore, \$489.677 rounded to the nearest cent (or to two decimal places) is \$489.68.

- (iii) *Rounding \$39.9985 to the nearest cent*

- Identify the rounding digit in the hundredths place: \$39.9985.
- The digit to the immediate right of the rounding digit is greater than 5; therefore, increase the value of the rounding digit by one, from 9 to 10, carrying the one to the tenths place, then to the ones, then to the tens to increase 3 to 4. Finally, drop all the digits that are to the right of the hundredths place.

Therefore, \$39.9985 rounded to the nearest cent (or to two decimal places) is \$40.00.