ANSEL C. UGURAL ▪ SAUL K. FENSTER

Advanced Mechanics of Materials and Applied Elasticity

SIXTH EDITION

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Preface

INTRODUCTION

Advanced Mechanics of Materials and Applied Elasticity, Sixth Edition, is an outgrowth of classroom notes prepared in connection with advanced undergraduate and first-year graduate courses in the mechanics of solids and elasticity. It is designed to satisfy the requirements of courses subsequent to an elementary treatment of the strength of materials. In addition to its applicability to aeronautical, civil, and mechanical engineering and to engineering mechanics curricula, the text is useful to practicing engineers. Emphasis is given to *numerical techniques* (which lend themselves to computerization) in the solution of problems resisting *analytical treatment*. The attention devoted to numerical solutions is not intended to deny the value of classical analysis, which is given a rather full treatment. Instead, the coverage provided here seeks to fill what we believe to be a void in the world of textbooks.

We have attempted to present a balance between the theory necessary to gain insight into the mechanics, but which can often offer no more than crude approximations to real problems because of simplifications related to geometry and conditions of loading, and numerical solutions, which are so useful in presenting stress analysis in a more realistic setting. This text emphasizes those aspects of theory and application that prepare a student for more advanced study or for professional practice in design and analysis.

The theory of elasticity plays three important roles in the text. First, it provides exact solutions where the configurations of loading and boundary are relatively simple. Second, it provides a check on the limitations of the mechanics of materials approach. Third, it serves as the basis of approximate solutions employing numerical analysis.

To make the text as clear as possible, the fundamentals of the mechanics of materials are addressed as necessary. The physical significance of the solutions and practical applications are also emphasized. In addition, we have made a special effort to illustrate important principles and applications with numerical examples. Consistent with announced national policy, problems are included in the text in which the physical quantities are expressed in the International System of Units (SI). All important quantities are defined in both SI and U.S. Customary System (USCS) of units. A sign convention, consistent with vector mechanics, is employed throughout for loads, internal forces, and stresses. This convention conforms to that used in most classical strength of materials and elasticity texts, as well as to that most often employed in the numerical analysis of complex structures.

ORGANIZATION OF THE TEXT

Because of its extensive subdivision into a variety of topics and use of alternative methods of analysis, this text provides great flexibility for instructors when choosing assignments to cover courses of varying length and content. Most chapters are substantially self-contained, so the order of presentation can be smoothly altered to meet an instructor's preference. Ideally, Chapters 1 and 2, which address the analysis of basic concepts, should be studied first. The emphasis placed on the treatment of two-dimensional problems in elasticity (Chapter 3) may then differ according to the scope of the course.

This sixth edition of *Advanced Mechanics of Materials and Applied Elasticity* seeks to preserve the objectives and emphases of the previous editions. Every effort has been made to provide a more complete and current text through the inclusion of new material dealing with the fundamental principles of stress analysis and design: stress concentrations, contact stresses, failure criteria, fracture mechanics, compound cylinders, finite element analysis (FEA), energy and variational methods, buckling of stepped columns, common shell types, case studies in analysis and design, and MATLAB solutions. The entire text has been reexamined, and many improvements have been made throughout by a process of elimination and rearrangement. Some sections have been expanded to improve on previous expositions.

The references (identified in *brackets*), which are provided as an aid to those students who wish to pursue certain aspects of a subject in further depth, have been updated and listed at the end of each chapter. We have resisted the temptation to increase the material covered except where absolutely necessary. Nevertheless, we have added a number of illustrative examples and problems important in engineering practice and design. Extra care has been taken in the presentation and solution of the sample problems. All the problem sets have been reviewed and checked to ensure both their clarity and their numerical accuracy. Most changes in subject-matter coverage were prompted by the suggestions of faculty familiar with earlier editions.

In this sixth edition, we have maintained the previous editions' clarity of presentation, simplicity as the subject permits, unpretentious depth, an effort to encourage intuitive understanding, and a shunning of the irrelevant. In this context, as throughout, emphasis is placed on the use of fundamentals to help build students' understanding and ability to solve the more complex problems.

SUPPLEMENTS

The book is accompanied by a comprehensive instructor's *Solutions Manual*. Written and class tested, it features complete solutions to all problems in the text. Answers to selected problems are given at the end of the book. The password-protected Solutions Manual is available for adopters at the Pearson Instructor Resource Center, pearsonhighered.com/irc.

Optional Material is also available from the Pearson Resource Center, pearsonhighered.com/irc. This material includes PowerPoint slides of figures and tables, and solutions using MATLAB for a variety of sample problems of practical importance. The book, however, is independent of any software package.

> Register your copy of *Advanced Mechanics of Materials and Applied Elasticity, Sixth Edition*, on the InformIT site for convenient access to updates and corrections as they become available. To start the registration process, go to informit.com/register and log in or create an account. Enter the product ISBN (9780134859286) and click Submit. Look on the Registered Products tab for an Access Bonus Content link next to this product, and follow that link to access any available bonus materials. If you would like to be notified of exclusive offers on new editions and updates, please check the box to receive email from us.

Acknowledgments

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Accuracy checking of the problems, proofreading, and typing of the *Solutions Manual* were done expertly by my former student, Dr. Youngjin Chung. Also contributing considerably to this volume with typing new inserts, scanning some figures, limited proofreading, and cover design was Errol A. Ugural. Their hard work is much appreciated. I am deeply indebted to my colleagues who have found the text useful through the years, as well as to Laura Lewin, executive editor at Pearson, who encouraged development of this edition. Lastly, I am very thankful for the support and understanding of my wife Nora, daughter Aileen, and son Errol during preparation of this book.

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Ansel C. Ugural, Ph.D., has been a research and visiting professor of mechanical and civil engineering at the New Jersey Institute of Technology. He has taught in the engineering mechanics department at the University of Wisconsin. Dr. Ugural also served as chairman and tenured professor of mechanical engineering at Fairleigh Dickinson University for twenty years. He has considerable and diverse industrial experience in both full-time and consulting capacities as a design, development, and research engineer.

Professor Ugural earned his MS in mechanical engineering and PhD in engineering mechanics from the University of Wisconsin–Madison. Dr. Ugural was a National Science Foundation (NSF) fellow. He has been a member of several professional societies, including the American Society of Mechanical Engineers and the American Society of Engineering Education. He is also listed in *Who's Who in Engineering*.

Dr. Ugural is the author of several books, including *Mechanics of Materials*; *Mechanical Design: An Integrated Approach*; *Mechanical Design of Machine Components*; *Stresses in Beams, Plates, and Shells*; and *Plates and Shells: Theory and Analysis*. Most of these texts have been translated into Korean, Chinese, and Portuguese. In addition, Professor Ugural has published numerous articles in trade and professional journals.

Saul K. Fenster, Ph.D., served as president and tenured professor at New Jersey Institute of Technology for more than two decades. In addition, he has held varied positions at Fairleigh Dickinson University and taught at the City University of New York. His experience includes membership on a number of corporate boards and economic development commissions. Fenster is a fellow of the American Society of Mechanical Engineers, the American Society for Engineering Education, and the American Society for Manufacturing Engineers. He is coauthor of a text on mechanics.

Symbols

Roman Letters

A	area
B	width
C	carryover factor, torsional rigidity
c	distance from neutral axis to outer fiber
D	distribution factor, flexural rigidity of plate
$[D]$	elasticity matrix
d	diameter, distance
E	modulus of elasticity in tension or compression
E_s	modulus of plasticity or secant modulus
E_t	tangent modulus
e	dilatation, distance, eccentricity
$\{F\}$	nodal force matrix of bar and beam finite elements
F	body force per unit volume, concentrated force
f	coefficient of friction
$\{f\}$	displacement function of finite element
G	modulus of elasticity in shear or modulus of rigidity
g	acceleration of gravity (≈9.81 m/s^2)
h	depth of beam, height, membrane deflection, mesh width
I	moment of inertia of area, stress invariant
J	polar moment of inertia of area, strain invariant
K	bulk modulus, spring constant of an elastic support, stiffness factor, thermal conductivity, fatigue factor, strength coefficient, stress concentration factor
$[K]$	stiffness matrix of whole structure
k	constant, modulus of elastic foundation, spring constant
$[k]$	stiffness matrix of finite element
L	length, span
l, m, n	direction cosines
M	moment
M_{xy}	twisting moment in plates
m	moment caused by unit load

N	fatigue life (cycles), force
n	factor of safety, number, strain hardening index
P	concentrated force
p	distributed load per unit length or area, pressure, stress resultant
Q	first moment of area, heat flow per unit length, shearing force
$\{Q\}$	nodal force matrix of two-dimensional finite element
R	radius, reaction
r	radius, radius of gyration
r, θ	polar coordinates
S	elastic section modulus, shear center
s	distance along a line or a curve
T	temperature, twisting couple or torque
t	thickness
U	strain energy
U_o	strain energy per unit volume
U^*	complementary energy
u, υ, w	components of displacement
V	shearing force, volume
υ	velocity
W	weight, work
x, y, z	rectangular coordinates
Z	plastic section modulus

Greek Letters

α	angle, coefficient of thermal expansion, form factor for shear
β	numerical factor, angle
γ	shear strain, weight per unit volume or specific weight, angle
δ	deflection, finite difference operator, variational symbol, displacement
$\{\delta\}$	nodal displacement matrix of finite element
Δ	change of a function
ε	normal strain
θ	angle, angle of twist per unit length, slope
ν	Poisson's ratio
λ	axial load factor, Lamé constant
Π	potential energy
ρ	density (mass per unit volume), radius
σ	normal stress
τ	shear stress
ϕ	total angle of twist
Φ	stress function
ω	angular velocity
ψ	stream function

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CHAPTER 1

Analysis of Stress

1.1 INTRODUCTION

There are two major parts to this chapter. Review of some important fundamentals of statics and mechanics of solids, the concept of stress, modes of load transmission, the general sign convention for stress and force resultants that will be used throughout the book, and analysis and design principles are provided first. This is followed by treatment of changing the components of the state of stress given in one set of coordinate axes to any other set of rotated axes, as well as variation of stress within and on the boundaries of a load-carrying member. Plane stress and its transformation are of basic importance, since these conditions are most common in engineering practice. This chapter is therefore also a brief guide and introduction to the remainder of the text.

1.1.1 Mechanics of Materials and Theory of Elasticity

The basic structure of matter is characterized by nonuniformity and discontinuity attributable to its various subdivisions: molecules, atoms, and subatomic particles. Our concern in this text is not with the particulate structure, however. Instead, it will be assumed that the matter with which we are concerned is *homogeneous* and *continuously* distributed over its volume. There is the clear implication in such an approach that the smallest element cut from the body possesses the same properties as the body. Random fluctuations in the properties of the material are, therefore, of no consequence. This approach is that of *continuum mechanics*, in which solid elastic materials are treated as though they are continuous media rather than composed of discrete molecules. Of the states of matter, we are here concerned only with the solid, with its ability to maintain its shape without the need of a container and to resist continuous shear, tension, and compression.

In contrast with rigid-body statics and dynamics, which treat the external behavior of bodies (that is, the equilibrium and motion of bodies without regard to small deformations associated with the application of load), the mechanics of solids is concerned with the relationships of external effect (forces and moments) to internal stresses and strains. Two notable approaches used in solid mechanics are the *mechanics of materials* or *elementary theory* (also called *technical theory*) and the *theory of elasticity*. The mechanics of materials focuses mainly on the more or less approximate solutions of practical problems. The theory of elasticity concerns itself largely with more mathematical analysis to determine the "exact" stress and strain distributions in a loaded body. The difference between these approaches lies primarily in the nature of the simplifying assumptions used, described in Section 3.1.

External forces acting on a body may be classified as *surface forces* and *body forces*. A surface force is of the *concentrated* type when it acts at a point; a surface force may also be distributed *uniformly* or *nonuniformly* over a finite area. Body forces are associated with the mass rather than the surfaces of a body, and are distributed throughout the volume of a body. Gravitational, magnetic, and inertia forces are all body forces. They are specified in terms of force per unit volume. All forces acting on a body, including the reactive forces caused by supports and body forces, are considered to be *external forces*. *Internal forces* are the forces that hold together the particles forming the body. Unless otherwise stated, we assume in this text that body forces can be neglected and that forces are applied steadily and slowly. The latter is referred to as *static loading*.

In the International System of Units (SI), force is measured in newtons (N). Because the newton is a small quantity, the kilonewton (kN) is often used in practice. In the U.S. Customary System (USCS), force is expressed in pounds (lb) or kilopounds (kips). We define all important quantities in both systems of units. However, in numerical examples and problems, SI units are used throughout the text consistent with international convention. (Table D.2 compares the two systems.)

1.1.2 Historical Development

The study of the behavior of members in tension, compression, and bending began with Leonardo da Vinci (1452–1519) and Galileo Galilei (1564–1642). For a proper understanding, however, it was necessary to establish accurate experimental description of a material's properties. Robert Hooke (1615–1703) was the first to point out that a body is deformed subject to the action of a force. Sir Isaac Newton (1642–1727) developed the concepts of Newtonian mechanics that became key elements of the strength of materials.

Leonard Euler (1707–1783) presented the mathematical theory of columns in 1744. The renowned mathematician Joseph-Louis Lagrange (1736–1813) received credit for developing a partial differential equation to describe plate vibrations. Thomas Young (1773–1829) established a coefficient of elasticity, Young's modulus. The advent of railroads in the late 1800s provided the impetus for much of the basic work in this area. Many famous scientists and engineers, including Coulomb, Poisson, Navier, St. Venant, Kirchhoff, and Cauchy, were responsible for advances in mechanics of materials during the eighteenth and nineteenth centuries. The British physicist William Thomas Kelvin (1824–1907), better known by his knighted name, Sir Lord Kelvin, first demonstrated that torsional moments acting at the edges of plates could be decomposed into shearing forces. The prominent English mathematician Augustus Edward Hough Love (1863–1940) introduced simple analysis of shells, known as Love's approximate theory.

Over the years, most basic problems of solid mechanics had been solved. Stephan P. Timoshenko (1878–1972) made numerous original contributions to the field of applied mechanics and wrote pioneering textbooks on the mechanics of materials, theory of elasticity, and theory of elastic stability. The theoretical base for modern strength of materials had been developed by the end of the nineteenth century. Since that time, problems associated with the design of aircraft, space vehicles, and nuclear reactors have led to many studies of the more advanced phases of the subject. Consequently, the mechanics of materials is being expanded into the theories of elasticity and plasticity.

In 1956, Turner, Clough, Martin, and Topp introduced the *finite element method*, which permits the numerical solution of complex problems in solid mechanics in an economical way. Many contributions in this area are owed to Argyris and Zienkiewicz. The recent trend in the development is characterized by heavy reliance on high-speed computers and by the introduction of more rigorous theories. Numerical methods presented in Chapter 7 and applied in the subsequent chapters have clear application to computation by means of electronic digital computers. Research in the foregoing areas is ongoing, not only to meet demands for treating complex problems, but also to justify further use and limitations on which the theory of solid mechanics is based.

Although a widespread body of knowledge exists at present, mechanics of materials and elasticity remain fascinating subjects, as their areas of application continue to expand.* The literature dealing with various aspects of solid mechanics is voluminous. For those seeking more thorough treatment, selected references are identified in brackets and compiled at the end of each chapter.

1.2 SCOPE OF THE BOOK

As stated in the preface, this book is intended for advanced undergraduate and graduate engineering students as well as engineering professionals. To make the text as clear as possible, attention is given to the fundamentals of solid mechanics and chapter objectives. A special effort has been made to illustrate important principles and applications with numerical examples. Emphasis is placed on a thorough presentation of both classical topics in advanced mechanics of materials and applied elasticity and selected advanced topics. The physical behavior of members is first explained, and this behavior is then modeled to develop the theory.

The usual objective of the mechanics of materials and theory of elasticity is the examination of the load-carrying capacity of a body from three standpoints: *strength*, *stiffness*, and *stability.* Recall that these quantities relate, respectively, to the ability of a member to resist permanent deformation or fracture, to resist deflection, and to retain its equilibrium configuration. For instance, when loading produces an abrupt shape change of a member, instability occurs; similarly, an inelastic deformation or an excessive magnitude of deflection in a member will cause malfunction in normal service. Based on the *fundamental principles* (Section 1.3), these behaviors are discussed in later chapters for various types

*Historical reviews of the mechanics of materials and the theory of elasticity are given in Refs. 1.1 through 1.3.

of structural members. *Failure* by yielding and fracture of the materials under combined loading is taken up in detail in Chapter 4.

Our main concern is the analysis of stress and deformation within a loaded body, which is accomplished by application of one of the methods described in the next section. For this purpose, the analysis of loads is essential. A structure or machine cannot meet its expectations unless its design is based on realistic operating loads. The principal topics under the heading of *mechanics of solids* may be summarized as follows:

1. Analysis of the stresses and deformations within a body subject to a prescribed system of forces. This is accomplished by solving the governing equations that describe the stress and strain fields (theoretical stress analysis). It is often advantageous, where the shape of the structure or conditions of loading preclude a theoretical solution or where verification is required, to apply the laboratory techniques of experimental stress analysis.
2. Determination by theoretical analysis or by experiment of the limiting values of load that a structural element can sustain without suffering damage, failure, or compromise of function.
3. Determination of the body shape and selection of the materials that are most efficient for resisting a prescribed system of forces under specified conditions of operation, such as temperature, humidity, vibration, and ambient pressure. This is the design function.

The design function, item 3, clearly relies on the theoretical analyses results obtained via items 1 and 2; thus, this text focuses on those topics. In particular, emphasis is placed on the development of the equations and methods by which detailed analysis can be accomplished.

The ever-increasing industrial demand for more sophisticated structures and machines calls for a good grasp of the concepts of stress and strain and the behavior of materials—and a considerable degree of ingenuity. This text, at the very least, provides the student with the ideas and information necessary for an understanding of the advanced mechanics of solids and encourages use of the creative process based on that understanding. Complete, carefully drawn free-body diagrams are used to visualize the processes involved, though the subject matter can be learned best by solving problems of practical importance. A thorough grasp of fundamentals will prove of great value in attacking new and unfamiliar problems.

1.3 ANALYSIS AND DESIGN

Throughout this text, a fundamental procedure for analysis in solving mechanics of solids problems is used repeatedly. The complete analysis of load-carrying structural members by the **method of equilibrium** requires consideration of three conditions related to certain laws of forces, laws of material deformation, and geometric compatibility. These essential relationships, called the *basic principles of analysis*, are as follows:

1. Equilibrium conditions. The equations of equilibrium of forces must be satisfied throughout the member.

2. **Material behavior**. The stress–strain or *force-deformation relations* (for example, Hooke's law) must apply to the behavior of the material of which the member is constructed.
3. **Geometry of deformation**. The *compatibility conditions* of deformations must be satisfied: that is, each deformed portion of the member must fit together with adjacent portions. (Matter of compatibility is not always broached in mechanics of materials analysis.)

The stress and deformation obtained through the use of these three principles must conform to the conditions of loading imposed at the boundaries of a member. This circumstance is known as satisfying the **boundary conditions**. Applications of the preceding procedure are illustrated in the problems presented in this text. Note, however, that it is not always necessary to execute an analysis in the precise sequence of steps listed previously.

As an alternative to the equilibrium methods, the analysis of stress and deformation can be accomplished by employing **energy methods** (Chapter 10), which are based on the concept of strain energy. Both the equilibrium and the energy approaches can provide solutions of acceptable accuracy where configurations of loading and member shape are regular, and they can be used as the basis of **numerical methods** in the solution of more realistic problems.

Engineering design is the process of applying science and engineering techniques to define a structure or system in detail to allow its realization. The objective of a *mechanical design* procedure includes finding the proper materials, dimensions, and shapes of the members of a structure or machine so that they will support the prescribed loads and perform without failure. *Machine design* entails creating new or improved machines to accomplish specific purposes. Usually, *structural design* deals with any engineering discipline that requires a structural member or system.

Design is the essence, art, and intent of engineering. A good design satisfies performance, cost, and safety requirements. An *optimum design* is the best solution to a design problem within given restrictions. Efficiency of the optimization may be gaged by such criteria as minimum weight or volume, optimum cost, and any other standard deemed appropriate. When faced with a design problem characterized by many choices, a designer may often make decisions on the basis of past experience, so as to reduce the problem to a single variable. A solution to determine the optimum result becomes straightforward in such a situation.

A plan for satisfying a need usually includes preparation of individual preliminary design. Each *preliminary design* involves a thorough consideration of the loads and actions that the structure or machine must support. For each situation, an analysis is necessary. Design decisions—that is, choosing reasonable values of the safety factors and material properties—are significant in the preliminary design process. We note that the design of numerous structures, such as pressure vessels, space missiles, aircrafts, dome roofs, and bridge decks, is based on the theories of plates and shells. For example, a water storage tank can be satisfactorily designed using the shell-membrane theory (Section 13.12). By comparison, the design of a missile casing demands a more precise shell-bending theory so as to minimize weight and materials. Similarly, the design of a nozzle-to-cylinder junction in a nuclear reactor may necessitate an elaborate finite element analysis.

1.3.1 Role of Analysis in Design

This text provides an elementary treatment of the concept of "design to meet strength requirements" as those requirements relate to individual machine or structural components. That is, the geometric configuration and material of a component are preselected and the applied loads are specified. Then, the basic formulas for stress are employed to select members of adequate size in each case. The **role of analysis in design** may be observed best in examining the phases of a design process. The following is *rational procedure in the design* of a load-carrying member:

1. *Evaluate the most likely modes of failure of the member*. Failure criteria that predict the various modes of failure under anticipated conditions of service are discussed in Chapter 4.
2. *Determine the expressions relating applied loading to such effects as stress, strain, and deformation*. Often, the member under consideration and conditions of loading are so significant or so amenable to solution as to have been the subject of prior analysis. For these situations, textbooks, handbooks, journal articles, and technical papers are good sources of information. If the situation is unique, however, a mathematical derivation specific to the case at hand is required.
3. *Determine the maximum usable value of stress, strain, or energy*. This value is obtained either by reference to compilations of material properties or by experimental means such as simple tension test and is used in connection with the relationship derived in step 2.
4. *Select a design factor of safety*. This is to account for uncertainties in a number of aspects of the design, including those related to the actual service loads, material properties, or environmental factors. An important area of uncertainty is connected with the assumptions made in the analysis of stress and deformation. Also, we are not likely to have a secure knowledge of the stresses that may be introduced during machining, assembly, and shipment of the element.

The design factor of safety also reflects the consequences of failure—for example, the possibility that failure will result in loss of human life or injury or in costly repairs or danger to other components of the overall system. For these reasons, the design factor of safety is also sometimes called the *factor of ignorance*. The uncertainties encountered during the design phase may be of such magnitude as to lead to a design carrying extreme weight, volume, or cost penalties. It may then be advantageous to perform thorough tests or more exacting analysis rather to rely on overly large design factors of safety.

1.3.2 Selection of Factor of Safety

The *true factor of safety*, usually referred to simply as the factor of safety, can be determined only after the member is constructed and tested. This factor is the ratio of the maximum load that the member *can sustain* under severe testing without failure to the maximum load that is *actually* carried under normal service conditions (the working load). When a linear relationship exists between the load and the stress produced by the load, the *factor of safety n* may be expressed as

$$n = \frac{\text{maximum usable stress}}{\text{allowable stress}} \tag{1.1}$$

Maximum usable stress represents either the yield stress or the ultimate stress. The allowable stress is the working stress. The factor of safety must be greater than 1.0 if failure is to be avoided. Modern engineering design accounts for all possible environmental, loading, stress, and material conditions, leaving relatively few items of uncertainty to be covered by a factor of safety. Values for the factor of safety, selected by the designer on the basis of experience and judgment, range from approximately 1.25 to 4.

In the nuclear reactor industries, the safety factor is of prime significance in the face of many unknown effects; hence the factor of safety may be as high as 5. The use of a factor of safety in design is a reliable, time-proven approach. If this factor is properly selected, sound and safe designs are obtained by using it. For most applications, appropriate factors of safety are found in various construction and manufacturing codes. A concept closely related to safety factor is *reliability* defined as the statistical measure of the probability that a member will not fail in service [Ref. 1.4].

The procedure outlined in Section 1.3.1 is not always conducted in as formal a fashion as may have been implied in that discussion. In some design phases, one or more steps may be regarded as unnecessary or obvious on the basis of previous experience. Suffice it to say that complete design solutions are not unique, involve a consideration of many factors, and often require a trial-and-error process. Stress is just one consideration in design. Other phases of the design of components include the prediction of the deformation of a given component under given loading and the consideration of buckling. The methods of determining deformation are discussed in later chapters of this text. Note that analysis and design are closely related, and the examples and problems that appear throughout this book illustrate that connection.

We conclude this section with an appeal to the reader to exercise a degree of skepticism when applying formulas for which the limitations of use or the areas of applicability are uncertain. The relatively simple form of many formulas usually results from rather severe restrictions in the formula's derivation. These limitations may include simplified boundary conditions and shapes, limitations on stress and strain, and the neglect of certain complicating factors. Designers and stress analysts must be aware of such restrictions lest their work be of no value or, even worse, lead to dangerous inadequacies.

In this chapter, we focus on the state of *stress at a point* and the *variation of stress* throughout an elastic body. The latter is dealt with in Sections 1.8 and 1.16 and the former in the balance of the chapter.

1.3.3 Case Studies

A general *case study* in *analysis* may move step by step through the problem formulation and solution stages, as outlined in Appendix A. The basic geometry and loading on a member must be selected before any analysis can be done. For example, the stress that would occur in a bar under a load would depend on whether the loading gives rise to tension, transverse shear, direct shear, torsion, bending, or contact stresses. In this case, *uniform stress* patterns may be more efficient at carrying the load than others. Therefore, by carefully studying the types of loads and stress patterns that can arise in structures, some insight can be gained into improved shapes and orientations of components. A few case studies introduced in this text involve situations encountered during the analysis and design of structural members.

1.4 CONDITIONS OF EQUILIBRIUM

A *structure* is a unit consisting of interconnected members supported in such a way that it is capable of carrying loads in static equilibrium. Structures are of four general types: frames, trusses, machines, and thin-walled (plate and shell) structures. *Frames* and *machines* are structures containing multiforce members. The former support loads and are usually stationary, fully restrained structures. The latter transmit and modify forces (or power) and always contain moving parts. The *truss* provides both a practical and an economical solution, particularly in the design of bridges and buildings. When the truss is loaded at its joints, the only force in each member is an axial force, either tensile or compressive.

The analysis and design of structural and machine components require a knowledge of the distribution of forces within such members. Fundamental concepts and conditions of static equilibrium provide the necessary background for the determination of internal as well as external forces. In Section 1.6, we shall see that components of internal-forces resultants have special meaning in terms of the type of deformations they cause, as applied, for example, to slender members. The surface forces that develop at support points of a structure, which are called *reactions*, equilibrate the effects of the applied loads on the structures.

The **equilibrium** of forces is the state in which the forces applied on a body are in balance. Newton's first law states that if the resultant force acting on a particle (the simplest body) is zero, the particle will remain at rest or will move with constant velocity. Statics is concerned essentially with the case where the particle or body remains at rest. A complete free-body diagram is essential in the solution of equilibrium.

Let us consider the equilibrium of a body in space. In this three-dimensional case, the **conditions of equilibrium** require the satisfaction of the following **equations of statics**:

$$\begin{aligned} \Sigma F_x &= 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \\ \Sigma M_x &= 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0 \end{aligned} \tag{1.2}$$

These equations state that the sum of all forces acting on a body in any direction must be zero; the sum of all moments about any axis must be zero.

In a *planar problem*, where all forces act in a single (xy) plane, there are only three independent equations of statics:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0 \tag{1.3}$$

That is, the sum of all forces in any (x, y) directions must be zero, and the resultant moment about axis z or any point A in the plane must be zero. By replacing a force summation with an equivalent moment summation in Eqs. (1.3), the following *alternative* sets of conditions are obtained:

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0 \tag{1.4a}$$

provided that the line connecting the points A and B *is not* perpendicular to the x axis, or

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0 \tag{1.4b}$$

if points A, B, and C are *not* collinear. Clearly, the judicious selection of points for taking moments can often simplify the algebraic computations.

A structure is *statically determinate* when all forces on its members can be found by using only the conditions of equilibrium. If there are more unknowns than available equations of statics, the problem is called *statically indeterminate*. The degree of *static indeterminacy* is equal to the difference between the number of unknown forces and the number of relevant equilibrium conditions. Any reaction that is in excess of those that can be obtained by statics alone is termed *redundant*. Thus, the number of redundants is the same as the degree of indeterminacy.

1.5 DEFINITION AND COMPONENTS OF STRESS

Stress and strain are most important concepts for a comprehension of the mechanics of solids. They permit the mechanical behavior of load-carrying components to be described in terms fundamental to the engineer. Both the analysis and the design of a given machine or structural element involve the determination of stress and material stress–strain relationships. The latter is taken up in Chapter 2.

Consider a body in equilibrium subject to a system of external forces, as shown in Fig. 1.1a. Under the action of these forces, internal forces are developed within the body. To examine these forces at some interior point Q, we use an imaginary plane to cut the body at a section *a–a* through Q, dividing the body into two parts. As the forces acting on the entire body are in equilibrium, the forces acting on one part alone must be in

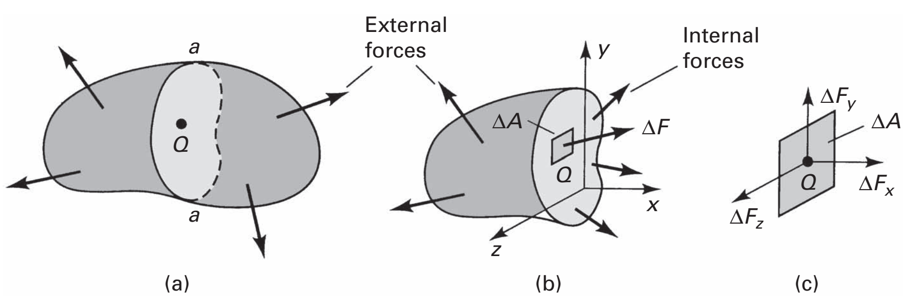

FIGURE 1.1. *Method of sections: (a) sectioning of a loaded body; (b) free body with external and internal forces; (c) enlarged area* ΔA *with components of the force* $\boldsymbol{\Delta F}$.