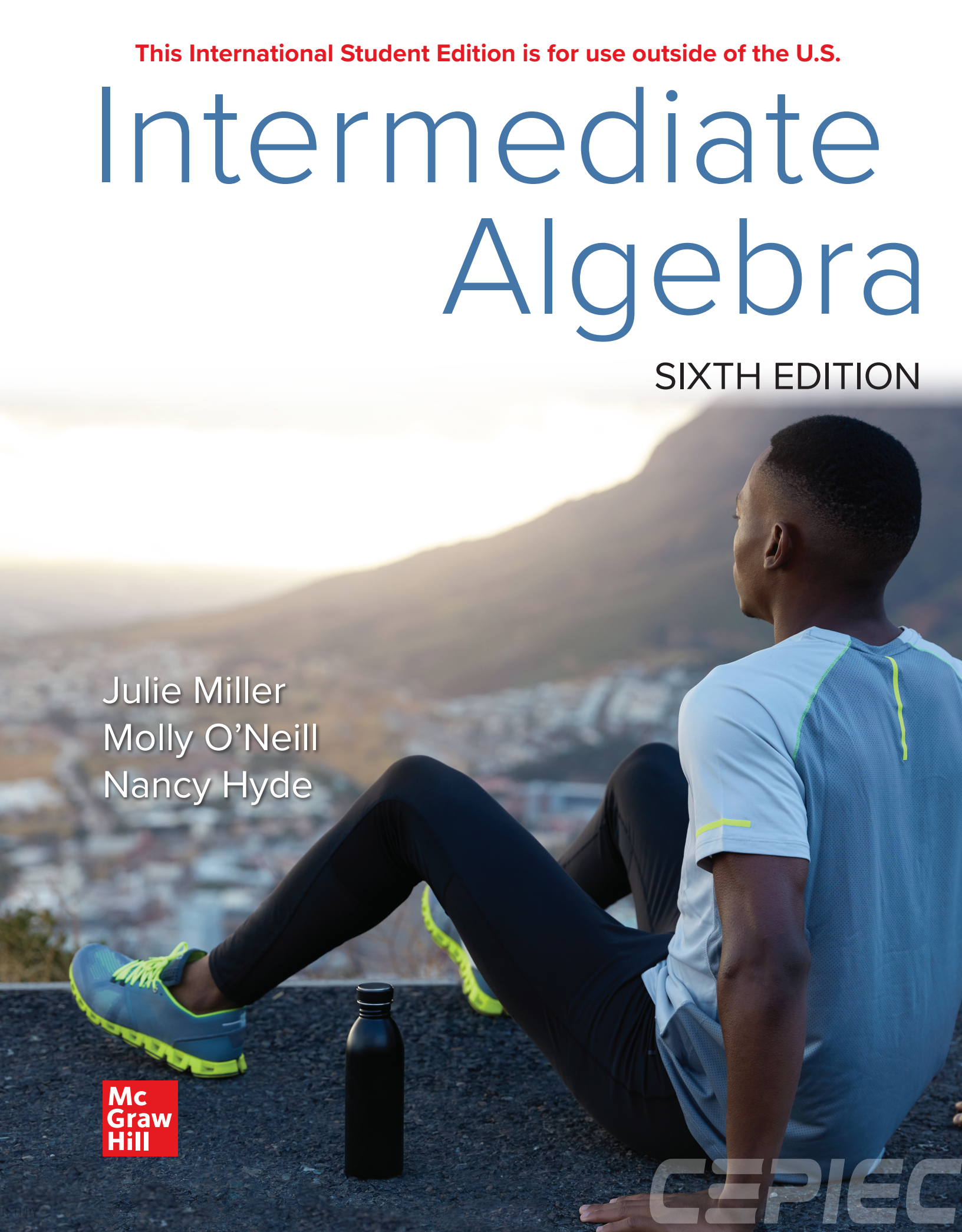
This International Student Edition is for use outside of the U.S.

# Intermediate Algebra

SIXTH EDITION

Julie Miller
Molly O'Neill
Nancy Hyde



Mc Graw Hill

# Intermediate Algebra

# Intermediate Algebra

SIXTH EDITION

**Julie Miller**
*Professor Emerita,*
*Daytona State College*

**Molly O'Neill**
*Professor Emerita,*
*Daytona State College*

**Nancy Hyde**
*Professor Emerita,*
*Broward College*

INTERMEDIATE ALGEBRA



This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 LWI 26 25 24 23 22 21

ISBN 978-1-264-41875-6
MHID 1-264-41875-2

Cover Image: ©*WAYHOME studio/Shutterstock*

All credits appearing on page or at the end of the book are considered to be an extension of the copyright page.

The Internet addresses listed in the text were accurate at the time of publication. The inclusion of a website does not indicate an endorsement by the authors or McGraw Hill LLC, and McGraw Hill LLC does not guarantee the accuracy of the information presented at these sites.

mheducation.com/highered

# Letter from the Authors

Dear Colleagues,

Across the country, Developmental Math courses are in a state of flux, and we as instructors are at the center of it all. As many of our institutions are grappling with the challenges of placement, retention, and graduation rates, we are on the front lines with our students—supporting all of them in their educational journey.

## Flexibility—No Matter Your Course Format!

The three of us each teach differently, as do many of our current users. The Miller/O'Neill/Hyde series is designed for successful use in a variety of course formats, both traditional and modern—classroom lecture settings, flipped classrooms, hybrid classes, and online-only classes.

## Ease of Instructor Preparation

We've all had to fill in for a colleague, pick up a last-minute section, or find ourselves running across campus to yet a different course. The Miller/O'Neill/Hyde series is carefully designed to support instructors teaching in a variety of different settings and circumstances. Experienced, senior faculty members can draw from a massive library of static and algorithmic content found in ALEKS to meticulously build assignments and assessments sharply tailored to individual student needs. Newer instructors and part-time adjunct instructors, on the other hand, will find support through a wide range of digital resources and prebuilt assignments ready to go on Day One. With these tools, instructors with limited time to prepare for class can still facilitate successful student outcomes.

Many instructors want to incorporate discovery-based learning and groupwork into their courses but don't have time to write or find quality materials. Each section of the text has numerous discovery-based activities that we have tested in our own classrooms. These are found in the text and Student Resource Manual along with other targeted worksheets for additional practice and materials for a student portfolio.

## Student Success—Now and in the Future

Too often our math placement tests fail our students, which can lead to frustration, anxiety, and often withdrawal from their education journey. We encourage you to learn more about ALEKS Placement, Preparation, and Learning (ALEKS PPL), which uses adaptive learning technology to place students appropriately. No matter the skills they come in with, the Miller/O'Neill/Hyde series provides resources and support that uniquely position them for success in that course and for their next course. Whether they need a brush-up on their basic skills, ADA supportive materials, or advanced topics to help them cross the bridge to the next level, we've created a support system for them.

We hope you are as excited as we are about the series and the supporting resources and services that accompany it. Please reach out to any of us with any questions or comments you have about our texts.

Julie Miller  Molly O'Neill  Nancy Hyde

# About the Authors

Photo courtesy of Molly O'Neill

**Julie Miller** is from Daytona State College, where she taught developmental and upper-level mathematics courses for 20 years. Prior to her work at Daytona State College, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a Bachelor of Science in Applied Mathematics from Union College in Schenectady, New York, and a Master of Science in Mathematics from the University of Florida. In addition to this textbook, she has authored textbooks for college algebra, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

"My father is a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I can remember using graph paper to plot data points for his experiments and doing simple calculations. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I'd like to see math come alive for my students."

*—Julie Miller*

**Molly O'Neill** is also from Daytona State College, where she taught for 22 years in the School of Mathematics. She has taught a variety of courses from developmental mathematics to calculus. Before she came to Florida, Molly taught as an adjunct instructor at the University of Michigan–Dearborn, Eastern Michigan University, Wayne State University, and Oakland Community College. Molly earned a Bachelor of Science in Mathematics and a Master of Arts and Teaching from Western Michigan University in Kalamazoo, Michigan. Besides this textbook, she has authored several course supplements for college algebra, trigonometry, and precalculus and has reviewed texts for developmental mathematics.

"I differ from many of my colleagues in that math was not always easy for me. But in seventh grade I had a teacher who taught me that if I follow the rules of mathematics, even I could solve math problems. Once I understood this, I enjoyed math to the point of choosing it for my career. I now have the greatest job because I get to do math every day and I have the opportunity to influence my students just as I was influenced. Authoring these texts has given me another avenue to reach even more students."

*—Molly O'Neill*

**Nancy Hyde** served as a full-time faculty member of the Mathematics Department at Broward College for 24 years. During this time she taught the full spectrum of courses from developmental math through differential equations. She received a Bachelor of Science in Math Education from Florida State University and a Master's degree in Math Education from Florida Atlantic University. She has conducted workshops and seminars for both students and teachers on the use of technology in the classroom. In addition to this textbook, she has authored a graphing calculator supplement for *College Algebra*.

"I grew up in Brevard County, Florida, where my father worked at Cape Canaveral. I was always excited by mathematics and physics in relation to the space program. As I studied higher levels of mathematics I became more intrigued by its abstract nature and infinite possibilities. It is enjoyable and rewarding to convey this perspective to students while helping them to understand mathematics."

*—Nancy Hyde*

## Dedication

To Our Students

Julie Miller ⚜ Molly O'Neill ⚜ Nancy Hyde

# The Miller/O'Neill/Hyde Developmental Math Series

Julie Miller, Molly O'Neill, and Nancy Hyde originally wrote their developmental math series because students were entering their College Algebra course underprepared. The students were not mathematically mature enough to understand the concepts of math, nor were they fully engaged with the material. The authors began their developmental mathematics offerings with Intermediate Algebra to help bridge that gap. This in turn evolved into several series of textbooks from Prealgebra through Precalculus to help students at all levels before Calculus.

What sets all of the Miller/O'Neill/Hyde series apart is that they address course content through an author-created digital package that maintains a consistent voice and notation throughout the program. This consistency—in videos, PowerPoints, Lecture Notes, and Integrated Video and Study Guides—coupled with the power of ALEKS, ensures that students master the skills necessary to be successful in Developmental Math through Precalculus and prepares them for the Calculus sequence.

**Developmental Math Series**

*The Developmental Math series is traditional in approach, delivering a purposeful balance of skills and conceptual development. It places a strong emphasis on conceptual learning to prepare students for success in subsequent courses.*

Basic College Mathematics, Third Edition
Prealgebra, Third Edition
Prealgebra & Introductory Algebra, Second Edition
Beginning Algebra, Sixth Edition
Beginning & Intermediate Algebra, Sixth Edition
Intermediate Algebra, Sixth Edition
Developmental Mathematics: Prealgebra, Beginning Algebra, & Intermediate Algebra, Second Edition

**The Miller/Gerken College Algebra/Precalculus Series**

*The Precalculus series serves as the bridge from Developmental Math coursework to future courses by emphasizing the skills and concepts needed for Calculus.*

College Algebra with Corequisite Support, First Edition
College Algebra, Second Edition
College Algebra and Trigonometry, First Edition
Precalculus, First Edition

# Acknowledgments

The author team most humbly would like to thank all the people who have contributed to this project and the Miller/O'Neill/Hyde Developmental Math series as a whole.

First and foremost, our utmost gratitude to Sarah Alamilla for her close partnership, creativity, and collaboration throughout this revision. Special thanks to our team of digital contributors for their thousands of hours of work: to Kelly Jackson, Jody Harris, Lizette Hernandez Foley, Lisa Rombes, Kelly Kohlmetz, and Leah Rineck for their devoted work. To Donna Gerken: thank you for the countless grueling hours working through spreadsheets to ensure thorough coverage of our content in ALEKS. To our digital authors, Linda Schott, Michael Larkin, and Alina Coronel: thank you for digitizing our content so it could be brought into ALEKS. We also offer our sincerest appreciation to the outstanding video talent: Alina Coronel, Didi Quesada, Tony Alfonso, and Brianna Ashley. So many students have learned from you! To Jennifer Blue, Carey Lange, John Murdzek, and Kevin Campbell: thank you so much for ensuring accuracy in our manuscripts.

We also greatly appreciate the many people behind the scenes at McGraw Hill without whom we would still be on page 1. To Megan Platt, our product developer: thank you for being our help desk and handling all things math, English, and editorial. To Brittney Merriman and Jennifer Morales, our portfolio managers and team leaders: thank you so much for leading us down this path. Your insight, creativity, and commitment to our project has made our job easier.

To the marketing team, Michele McTighe, Noah Evans, and Mary Ellen Rahn: thank you for your creative ideas in making our books come to life in the market. Thank you as well to Debbie McFarland, Justin Washington, and Sherry Bartel for continuing to drive our long-term content vision through their market development efforts. And many thanks to the team at ALEKS for creating its spectacular adaptive technology and for overseeing the quality control.

To the production team: Jane Mohr, David Hash, Lorraine Buczek, and Sandy Ludovissy—thank you for making the manuscript beautiful and for keeping the unruly authors on track. To our copyeditor Kevin Campbell and proofreader John Murdzek, who have kept a watchful eye over our manuscripts—the two of you are brilliant. To our compositor Manvir Singh and his team at Aptara, you've been a dream to work with. And finally, to Kathleen McMahon and Caroline Celano, thank you for supporting our projects for many years and for the confidence you've always shown in us.

Most importantly, we give special thanks to the students and instructors who use our series in their classes.

Julie Miller
Molly O'Neill
Nancy Hyde

# Contents

# To the Student

Take a deep breath and know that you aren't alone. Your instructor, fellow students, and we, your authors, are here to help you learn and master the material for this course and prepare you for future courses. You may feel like math just isn't your thing, or maybe it's been a long time since you've had a math class—that's okay!

We wrote the text and all the supporting materials with you in mind. Most of our students aren't really sure how to be successful in math, but we can help with that.

As you begin your class, we'd like to offer some specific suggestions:

1. **Attend class.** Arrive on time and be prepared. If your instructor has asked you to read prior to attending class—do it. How often have you sat in class and thought you understood the material, only to get home and realize you don't know how to get started? By reading and trying a couple of Skill Practice exercises, which follow each example, you will be able to ask questions and gain clarification from your instructor when needed.
2. **Be an *active* learner.** Whether you are at lecture, watching an author lecture or exercise video, or are reading the text, pick up a pencil and work out the examples given. Math is learned only by doing; we like to say, "Math is not a spectator sport." If you like a bit more guidance, we encourage you to use the Integrated Video and Study Guide. It was designed to provide structure and note-taking for lectures and while watching the accompanying videos.
3. **Schedule time to do some math every day.** Exercise, foreign language study, and math are three things that you must do every day to get the results you want. If you are used to cramming and doing all of your work in a few hours on a weekend, you should know that even mathematicians start making silly errors after an hour or so! Check your answers. Skill Practice exercises all have the answers at the bottom of that page. Odd-numbered exercises throughout the text have answers in the back of the text. If you didn't get it right, don't throw in the towel. Try again, revisit an example, or bring your questions to class for extra help.
4. **Prepare for quizzes and exams.** Each chapter has a set of Chapter Review Exercises at the end to help you integrate all of the important concepts. In addition, there is a detailed Chapter Summary and a Chapter Test. If you use ALEKS, use all of the tools available within the program to test your understanding.
5. **Use your resources.** This text comes with numerous supporting resources designed to help you succeed in this class and in your future classes. Additionally, your instructor can direct you to resources within your institution or community. Form a student study group. Teaching others is a great way to strengthen your own understanding, and they might be able to return the favor if you get stuck.

We wish you all the best in this class and in your educational journey!

Julie Miller  Molly O'Neill  Nancy Hyde

# Student Guide to the Text

## Clear, Precise Writing

Learning from our own students, we have written this text in simple and accessible language. Our goal is to keep you engaged and supported throughout your coursework.

## Call-Outs

Just as your instructor will share tips and math advice in class, we provide call-outs throughout the text to offer tips and warn against common mistakes.

- Tip boxes offer additional insight into a concept or procedure.
- Avoiding Mistakes help fend off common student errors.
- For Review boxes positioned strategically throughout the text remind students of key skills relating to the current topic.

## Examples

- Each example is step-by-step, with thorough annotation to the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to give you a chance to test your understanding. You will find the answer at the bottom of the page—providing a quick check.

## Exercise Sets

Each type of exercise is built so you can successfully learn the materials and show your mastery on exams.

- **Activities for discovery-based learning** appear before the exercise sets to walk students through the concepts presented in each section of the text.
- **Study Skills Exercises** integrate your studies of math concepts with strategies for helping you grow as a student overall.
- **Vocabulary and Key Concept Exercises** check your understanding of the language and ideas presented within the section.
- **Prerequisite Review** exercises keep fresh your knowledge of math content already learned by providing practice with concepts explored in previous sections.
- **Concept Exercises** assess your comprehension of the specific math concepts presented within the section.
- **Mixed Exercises** evaluate your ability to successfully complete exercises that combine multiple concepts presented within the section.
- **Expanding Your Skills** challenge you with advanced skills practice exercises around the concepts presented within the section.
- **Problem Recognition Exercises** appear in strategic locations in each chapter of the text. These will require you to distinguish between similar problem types and to determine what type of problem-solving technique to apply.
- **Technology Exercises** appear where appropriate.

## End-of-Chapter Materials

The features at the end of each chapter are perfect for reviewing before test time.

- **Section-by-section summaries** provide references to key concepts, examples, and vocabulary.
- **Chapter Review Exercises** provide additional opportunities to practice material from the entire chapter.
- **Chapter tests** are an excellent way to test your complete understanding of the chapter concepts.

# How Will Miller/O'Neill/Hyde Help Your Students *Get Better Results?*

## Clarity, Quality, and Accuracy

Julie Miller, Molly O'Neill, and Nancy Hyde know what students need to be successful in mathematics. Better results come from clarity in their exposition, quality of step-by-step worked examples, and accuracy of their exercise sets; but it takes more than just great authors to build a textbook series to help students achieve success in mathematics. Our authors worked with a strong team of mathematics instructors from around the country to ensure that the clarity, quality, and accuracy you expect from the Miller/O'Neill/Hyde series was included in this edition.

## Exercise Sets

Comprehensive sets of exercises are available for every student level. Julie Miller, Molly O'Neill, and Nancy Hyde worked with a board of advisors from across the country to offer the appropriate depth and breadth of exercises for your students. **Problem Recognition Exercises** were created to improve student performance while testing.

Practice exercise sets help students progress from skill development to conceptual understanding. Student tested and instructor approved, the Miller/O'Neill/Hyde exercise sets will help your students *get better results*.

- **Activities for Discovery-Based Learning**
- **Prerequisite Review Exercises**
- **Problem Recognition Exercises**
- **Skill Practice Exercises**
- **Study Skills Exercises**
- **Mixed Exercises**
- **Expanding Your Skills Exercises**
- **Vocabulary and Key Concepts Exercises**
- **Technology Exercises**

## Step-By-Step Pedagogy

This text provides enhanced step-by-step learning tools to help students *get better results*.

- **For Review** tips placed in the margin guide students back to related prerequisite skills needed for full understanding of course-level topics.
- **Worked Examples** provide an "easy-to-understand" approach, clearly guiding each student through a step-by-step approach to master each practice exercise for better comprehension.
- **TIPs** offer students extra cautious direction to help improve understanding through hints and further insight.
- **Avoiding Mistakes** boxes alert students to common errors and provide practical ways to avoid them. Both of these learning aids will help students get better results by showing how to work through a problem using a clearly defined step-by-step methodology that has been class tested and student approved.

# Get Better Results

## *Formula for Student Success*

### Step-by-Step Worked Examples

- Do you get the feeling that there is a disconnect between your students' class work and homework?
- Do your students have trouble finding worked examples that match the practice exercises?
- Do you prefer that your students see examples in the textbook that match the ones you use in class?

**Miller/O'Neill/Hyde's *Worked Examples* offer a clear, concise methodology that replicates the mathematical processes used in the authors' classroom lectures.**

**Example 1** Determining the Order of a Matrix

Determine the order of each matrix.

**a.** $\begin{bmatrix} 2 & -4 & 1 \\ 5 & \pi & \sqrt{7} \end{bmatrix}$ **b.** $\begin{bmatrix} 1.9 \\ 0 \\ 7.2 \\ -6.1 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **d.** $[a \quad b \quad c]$

**Solution:**

**a.** This matrix has two rows and three columns. Therefore, it is a $2 \times 3$ matrix.

**b.** This matrix has four rows and one column. Therefore, it is a $4 \times 1$ matrix. A matrix with one column is called a **column matrix**.

**c.** This matrix has three rows and three columns. Therefore, it is a $3 \times 3$ matrix. A matrix with the same number of rows and columns is called a **square matrix**.

**d.** This matrix has one row and three columns. Therefore, it is a $1 \times 3$ matrix. A matrix with one row is called a **row matrix**.

**Skill Practice** Determine the order of the matrix.

**1.** $\begin{bmatrix} -5 & 2 \\ 1 & 3 \end{bmatrix}$ **2.** $[4 \quad -8]$ **3.** $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$ **4.** $\begin{bmatrix} 2 & -0.5 \\ -1 & 6 \end{bmatrix}$

### Classroom Examples

To ensure that the classroom experience also matches the examples in the text and the practice exercises, we have included references to even-numbered exercises to be used as Classroom Examples. These exercises are highlighted in the Practice Exercises at the end of each section.

**Example 5** Finding the *x*- and *y*-Intercepts of a Line

Given $2x + 4y = 8$, find the *x*- and *y*-intercepts. Then graph the equation.

**Solution:**

To find the *x*-intercept, substitute $y = 0$.

$$2x + 4y = 8$$
$$2x + 4(0) = 8$$
$$2x = 8$$
$$x = 4$$

The *x*-intercept is (4, 0).

To find the *y*-intercept, substitute $x = 0$.

$$2x + 4y = 8$$
$$2(0) + 4y = 8$$
$$4y = 8$$
$$y = 2$$

The *y*-intercept is (0, 2).

# Quality Learning Tools

## *For Review* Boxes

Throughout the text, just-in-time tips and reminders of prerequisite skills appear in the margin alongside the concepts for which they are needed. References to prior sections are given for cases where more comprehensive review is available earlier in the text.

**FOR REVIEW**

Recall that the sum of an expression and its opposite is zero. For example:

$4y + (-4y) = 0$

## TIP and Avoiding Mistakes Boxes

**TIP** and **Avoiding Mistakes** boxes have been created based on the authors' classroom experiences—they have also been integrated into the **Worked Examples.** These pedagogical tools will help students get better results by learning how to work through a problem using a clearly defined step-by-step methodology.

**Example 7** Simplifying a Radical Expression

Simplify. $\frac{7\sqrt{50}}{10}$

**Solution:**

$\frac{7\sqrt{50}}{10} = \frac{7\sqrt{25 \cdot 2}}{10}$ 25 is the greatest perfect square in the radicand.

$= \frac{7 \cdot 5\sqrt{2}}{10}$ Simplify the radical.

$= \frac{7 \cdot \overset{1}{5}\sqrt{2}}{\underset{2}{10}}$ Simplify the fraction to lowest terms.

$= \frac{7\sqrt{2}}{2}$

**Avoiding Mistakes**

The expression $\frac{7\sqrt{2}}{2}$ cannot be simplified further because one factor of 2 is in the radicand and the other is outside the radical.

**Skill Practice** Simplify.

9. $\frac{2\sqrt{300}}{30}$

### Avoiding Mistakes Boxes:

*Avoiding Mistakes* boxes are integrated throughout the textbook to alert students to common errors and how to avoid them.

**TIP:** When solving a literal equation for a specified variable, there is sometimes more than one way to express your final answer. This flexibility often presents difficulty for students. Students may leave their answer in one form, but the answer given in the text may look different. Yet both forms may be correct. To know if your answer is equivalent to the form given in the text, you must try to manipulate it to look like the answer in the book, a process called *form fitting*.

The literal equation from Example 4 can be written in several different forms. The quantity $(2A - b_2h)/h$ can be split into two fractions.

$$b_1 = \frac{2A - b_2h}{h} = \frac{2A}{h} - \frac{b_2h}{h} = \frac{2A}{h} - b_2$$

### TIP Boxes

Teaching tips are usually revealed only in the classroom. Not anymore! TIP boxes offer students helpful hints and extra direction to help improve understanding and provide further insight.

# Get Better Results

## Better Exercise Sets and Better Practice Yield Better Results

- Do your students have trouble with problem solving?
- Do you want to help students overcome math anxiety?
- Do you want to help your students improve performance on math assessments?

## Problem Recognition Exercises

*Problem Recognition Exercises* present a collection of problems that look similar to a student upon first glance, but are actually quite different in the manner of their individual solutions. Students sharpen critical thinking skills and better develop their "solution recall" to help them distinguish the method needed to solve an exercise—an essential skill in mathematics.

**Problem Recognition Exercises** were tested in the authors' developmental mathematics classes and were created to improve student performance on tests.

### Problem Recognition Exercises

#### Rational Equations vs. Expressions

**1. a.** Simplify. $\frac{3}{w-5}+\frac{10}{w^2-25}-\frac{1}{w+5}$

**b.** Solve. $\frac{3}{w-5}+\frac{10}{w^2-25}-\frac{1}{w+5}=0$

**c.** Identify each problem in parts (a) and (b) as either an equation or an expression.

**2. a.** Simplify. $\frac{x}{2x+4}+\frac{2}{3x+6}-1$

**b.** Solve. $\frac{x}{2x+4}+\frac{2}{3x+6}=1$

**c.** Identify each problem in parts (a) and (b) as either an equation or an expression.

For Exercises 3–20, first ask yourself whether the problem is an expression to simplify or an equation to solve. Then simplify or solve as indicated.

**3.** $\frac{2}{a^2+4a+3}+\frac{1}{a+3}$

**4.** $\frac{1}{c+6}+\frac{4}{c^2+8c+12}$

**5.** $\frac{7}{y^2-y-2}+\frac{1}{y+1}-\frac{3}{y-2}=0$

**6.** $\frac{3}{b+2}-\frac{1}{b-1}-\frac{5}{b^2+b-2}=0$

**7.** $\frac{x}{x-1}-\frac{12}{x^2-x}$

**8.** $\frac{3}{5t-20}+\frac{4}{t-4}$

# Get Better Results

## Student-Centered Applications

The Miller/O'Neill/Hyde Board of Advisors partnered with our authors to bring the *best applications* from every region in the country! These applications include real data and topics that are more relevant and interesting to today's student.

**92.** $^{99m}$Tc is a radionuclide of technetium that is widely used in nuclear medicine. Although its half-life is only 6 hr, the isotope is continuously produced via the decay of its longer-lived parent $^{99}$Mo (molybdenum-99), whose half-life is approximately 3 days. The $^{99}$Mo generators (or "cows") are sold to hospitals in which the $^{99m}$Tc can be "milked" as needed over a period of a few weeks. Once separated from its parent, the $^{99m}$Tc may be chemically incorporated into a variety of imaging agents, each of which is designed to be taken up by a specific target organ within the body. Special cameras, sensitive to the gamma rays emitted by the technetium, are then used to record a "picture" (similar in appearance to an X-ray film) of the selected organ.

Suppose a technician prepares a sample of $^{99m}$Tc-pyrophosphate to image the heart of a patient suspected of having had a mild heart attack. If the injection contains 10 millicuries (mCi) of $^{99m}$Tc at 1:00 P.M., then the amount of technetium still present is given by

$$T(t) = 10e^{-0.1155t}$$

where $t > 0$ represents the time in hours after 1:00 P.M. and $T(t)$ represents the amount of $^{99m}$Tc (in millicuries) still present.

**a.** How many millicuries of $^{99m}$Tc will remain at 4:20 P.M. when the image is recorded? Round to the nearest tenth of a millicurie.

**b.** How long will it take for the radioactive level of the $^{99m}$Tc to reach 2 mCi? Round to the nearest tenth of an hour.

## Activities

Each section of the text ends with an activity that steps the student through the major concepts of the section. The purpose of the activities is to promote active, discovery-based learning for the student. The implementation of the activities is flexible for a variety of delivery methods. For face-to-face classes, the activities can be used to break up lecture by covering the exercises intermittently during the class. For the flipped classroom and hybrid classes, students can watch the videos and try the activities. Then, in the classroom, the instructor can go over the activities or have the students compare their answers in groups. For online classes, the activities provide great discussion questions.

### Section 2.6 Activity

**A.1.** Given a set of ordered pairs, how can you determine whether the relation defines $y$ as a function of $x$?

For Exercises A.2–A.3, consider the given relation.

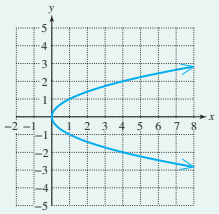
**a.** Do any two ordered pairs have the same $x$ value but different $y$ values?

**b.** Is the relation a function?

**A.2.** $\{(-5, 1), (3, 4), (-2, 6), (-5, 2), (0, -3)\}$

**A.3.** $\{(-1, 6), (2, 11), (8, 6), (-3, 1), (0.4, -0.5)\}$

**A.4.** **a.** For the graph given, draw a vertical line through the point (4, 2). Does the vertical line intersect the graph at any other point?




**b.** Does this graph define $y$ as a function of $x$?

**c.** Using this example, explain how the vertical line test is used to determine if a graph defines $y$ as a function of $x$.

**A.5.** Consider the equation $y = 2x + 1$.

**a.** If $x = 3$, what is the corresponding $y$ value?

**b.** Write the result of part (a) as an ordered pair $(x, y)$.

**A.6.** Consider the function defined by $f(x) = 2x + 1$.

**a.** Find $f(3)$. That is, evaluate the function for $x = 3$ by substituting 3 for $x$.

**b.** Write the result of part (a) as an ordered pair $(x, y)$.

**c.** Refer to Exercise A.5 and compare the results.

**A.7.** Given $g(x) = \dfrac{4}{x-1}$, find the function values if possible.

**a.** $g(2)$ **b.** $g(-3)$

**c.** $g(0)$ **d.** $g(1)$

**A.8.** Refer to $g(x) = \dfrac{4}{x-1}$ from Exercise A.7.

**a.** What value(s) of $x$ must be excluded from the domain of $g$? Why?

**b.** Write the domain of $g$ in interval notation.

**A.9.** Given $h(x) = \sqrt{x+2}$, find the function values if possible.

**a.** $h(-1)$ **b.** $h(2)$

**c.** $h(7)$ **d.** $h(-6)$

# Get Better Results

## *Additional Supplements*

### Lecture Videos Created by the Authors

Julie Miller began creating these lecture videos for her own students to use when they were absent from class. The student response was overwhelmingly positive, prompting the author team to create the lecture videos for their entire developmental math book series. In these videos, the authors walk students through the learning objectives using the same language and procedures outlined in the book. Students learn and review right alongside the author! Students can also access the written notes that accompany the videos.

### Integrated Video and Study Workbooks

The Integrated Video and Study Workbooks were built to be used in conjunction with the Miller/O'Neill/Hyde Developmental Math series online lecture videos. These new video guides allow students to consolidate their notes as they work through the material in the book, and they provide students with an opportunity to focus their studies on particular topics that they are struggling with rather than entire chapters at a time. Each video guide contains written examples to reinforce the content students are watching in the corresponding lecture video, along with additional written exercises for extra practice. There is also space provided for students to take their own notes alongside the guided notes already provided. By the end of the academic term, the video guides will not only be a robust study resource for exams, but will serve as a portfolio showcasing the hard work of students throughout the term.

### Dynamic Math Animations

The authors have constructed a series of animations to illustrate difficult concepts where static images and text fall short. The animations leverage the use of on-screen movement and morphing shapes to give students an interactive approach to conceptual learning. Some provide a virtual laboratory for which an application is simulated and where students can collect data points for analysis and modeling. Others provide interactive question-and-answer sessions to test conceptual learning.

### Exercise Videos

The authors, along with a team of faculty who have used the Miller/O'Neill/Hyde textbooks for many years, have created exercise videos for designated exercises in the textbook. These videos cover a representative sample of the main objectives in each section of the text. Each presenter works through selected problems, following the solution methodology employed in the text.

The video series is available online as part of ALEKS 360. The videos are closed-captioned for the hearing impaired and meet the Americans with Disabilities Act Standards for Accessible Design.

### Student Resource Manual

The *Student Resource Manual (SRM),* created by the authors, is a printable, electronic supplement available to students through ALEKS. Instructors can also choose to customize this manual and package it with their course materials. With increasing demands on faculty schedules, this resource offers a convenient means for both full-time and adjunct faculty to promote active learning and success strategies in the classroom.

This manual supports the series in a variety of different ways:

- Additional group activities developed by the authors to supplement what is already available in the text
- Discovery-based classroom activities written by the authors for each section
- Excel activities that not only provide students with numerical insights into algebraic concepts, but also teach simple computer skills to manipulate data in a spreadsheet

- Worksheets for extra practice written by the authors, including Problem Recognition Exercise Worksheets
- Lecture Notes designed to help students organize and take notes on key concepts
- Materials for a student portfolio

## Annotated Instructor's Edition

In the *Annotated Instructor's Edition* (*AIE*), answers to all exercises appear adjacent to each exercise in a color used *only* for annotations. The *AIE* also contains Instructor Notes that appear in the margin. These notes offer instructors assistance with lecture preparation. In addition, there are Classroom Examples referenced in the text that are highlighted in the Practice Exercises. Also found in the *AIE* are icons within the Practice Exercises that serve to guide instructors in their preparation of homework assignments and lessons.

## PowerPoints

The PowerPoints present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

## Test Bank

Among the supplements is a computerized test bank using the algorithm-based testing software TestGen® to create customized exams quickly. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank.

## ALEKS PPL: Pave the Path to Graduation with Placement, Preparation, and Learning

- **Success in College Begins with Appropriate Course Placement:** A student's first math course is critical to his or her success. With a unique combination of adaptive assessment and personalized learning, ALEKS Placement, Preparation, and Learning (PPL) accurately measures the student's math foundation and creates a personalized learning module to review and refresh lost knowledge. This allows the student to be placed and successful in the right course, expediting the student's path to complete their degree.
- **The Right Placement Creates Greater Value:** Students invest thousands of dollars in their education. ALEKS PPL helps students optimize course enrollment by avoiding courses they don't need to take and helping them pass the courses they do need to take. With more accurate student placement, institutions will retain the students that they recruit initially, increasing their recruitment investment and decreasing their DFW rates. Understanding where your incoming students are placing helps you to plan and develop course schedules and allocate resources efficiently.
- **See ALEKS PPL in Action:** http://bit.ly/ALEKSPPL

McGraw-Hill create®

McGraw-Hill Create allows you to select and arrange content to match your unique teaching style, add chapters from McGraw-Hill textbooks, personalize content with your syllabus or lecture notes, create a cover design, and receive your PDF review copy in minutes! Order a print or eBook for use in your course, and update your material as often as you'd like. Additional third-party content can be selected from a number of special collections on Create. Visit **McGraw-Hill Create** to browse Create Collections: **http://create.mheducation.com**.

# Our Commitment to Market Development and Accuracy

McGraw Hill's development process is an ongoing, market-oriented approach to building accurate and innovative print and digital products. We begin developing a series by partnering with authors who have a vision for positively impacting student success. Next, we share these ideas and the manuscript with instructors to review and provide feedback to ensure that the authors' ideas represent the needs of that discipline. Throughout multiple drafts, we help our authors to incorporate ideas and suggestions from reviewers to ensure that the series follows the pulse of today's classroom. With all editions, we commit to accuracy in the print text, supplements, and online platforms. In addition to involving instructors as we develop our content, we also perform accuracy checks throughout the various stages of development and production. Through our commitment to this process, we are confident that our series features content that has been thoughtfully developed and vetted to meet the needs of both instructors and students.

# Acknowledgments and Reviewers

Paramount to the development of this series was the invaluable feedback provided by the instructors from around the country who reviewed the manuscript or attended a market development event over the course of the several years the text was in development.

Maryann Faller, *Adirondack Community College*
Albert Miller, *Ball State University*
Debra Pearson, *Ball State University*
Patricia Parkison, *Ball State University*
Robin Rufatto, *Ball State University*
Melanie Walker, *Bergen Community College*
Robert Fusco, *Bergen Community College*
Latonya Ellis, *Bishop State Community College*
Ana Leon, *Bluegrass Community College & Technical College*
Kaye Black, *Bluegrass Community College & Technical College*
Barbara Elzey, *Bluegrass Community College & Technical College*
Cheryl Grant, *Bowling Green State University*
Beth Rountree, *Brevard College*
Juliet Carl, *Broward College*
Lizette Foley, *Broward College*
Angie Matthews, *Broward College*
Mitchel Levy, *Broward College*
Jody Harris, *Broward College*
Michelle Carmel, *Broward College*
Antonnette Gibbs, *Broward College*
Kelly Jackson, *Camden Community College*
Elizabeth Valentine, *Charleston Southern University*
Adedoyin Adeyiga, *Cheyney University of Pennsylvania*
Dot French, *Community College of Philadelphia*
Brad Berger, *Copper Mountain College*
Donna Troy, *Cuyamaca College*
Brianna Kurtz, *Daytona State College–Daytona Beach*
Jennifer Walsh, *Daytona State College–Daytona Beach*
Marc Campbell, *Daytona State College–Daytona Beach*
Richard Rupp, *Del Mar College*
Joseph Hernandez, *Delta College*
Randall Nichols, *Delta College*
Thomas Wells, *Delta College*
Paul Yun, *El Camino College*
Catherine Bliss, *Empire State College–Saratoga Springs*
Laurie Davis, *Erie Community College*
Linda Kuroski, *Erie Community College*
David Usinski, *Erie Community College*
Ron Bannon, *Essex County College*
David Platt, *Front Range Community College*
Alan Dinwiddie, *Front Range Community College*
Shanna Goff, *Grand Rapids Community College*
Betsy McKinney, *Grand Rapids Community College*
Cathy Gardner, *Grand Valley State University*
Jane Mays, *Grand Valley State University*
John Greene, *Henderson State University*
Fred Worth, *Henderson State University*
Ryan Baxter, *Illinois State University*
Angela Mccombs, *Illinois State University*
Elisha Van Meenen, *Illinois State University*
Teresa Hasenauer, *Indian River State College*

Tiffany Lewis, *Indian River State College*
Deanna Voehl, *Indian River State College*
Joe Jordan, *John Tyler Community College*
Sally Copeland, *Johnson County Community College*
Nancy Carpenter, *Johnson County Community College*
Susan Yellott, *Kilgore College*
Kim Miller, *Labette Community College*
Michelle Hempton, *Lansing Community College*
Michelle Whitmer, *Lansing Community College*
Kuen Lee, *Los Angeles Trade Tech*
Nic Lahue, *MCC-Longview Community College*
Jason Pallett, *MCC-Longview Community College*
Janet Wyatt, *MCC-Longview Community College*
Rene Barrientos, *Miami Dade College—Kendall*
Nelson De La Rosa, *Miami Dade College—Kendall*
Jody Balzer, *Milwaukee Area Technical College*
Shahla Razavi, *Mt. San Jacinto College*
Shawna Bynum, *Napa Valley College*
Tammy Ford, *North Carolina A & T University*
Ebrahim Ahmadizadeh, *Northampton Community College*
Christine Wetzel-Ulrich, *Northampton Community College*
Sharon Totten, *Northeast Alabama Community College*
Rodolfo Maglio, *Northeastern Illinios University*
Christine Copple, *Northwest State Community College*
Sumitana Chatterjee, *Nova Community College*
Charbel Fahed, *Nova Community College*
Ken Hirschel, *Orange County Community College*
Linda K. Schott, *Ozarks Technical Community College*
Matthew Harris, *Ozarks Technical Community College*
Daniel Kopsas, *Ozarks Technical Community College*
Andrew Aberle, *Ozarks Technical Community College*
Alan Papen, *Ozarks Technical Community College*
Angela Shreckhise, *Ozarks Technical Community College*
Jacob Lewellen, *Ozarks Technical Community College*
Marylynne Abbott, *Ozarks Technical Community College*
Jeffrey Gervasi, *Porterville College*
Stewart Hathaway, *Porterville College*
Lauran Johnson, *Richard Bland College*
Matthew Nickodemus, *Richard Bland College*
Cameron English, *Rio Hondo College*
Lydia Gonzalez, *Rio Hondo College*
Mark Littrell, *Rio Hondo College*
Matthew Pitassi, *Rio Hondo College*
Wayne Lee, *Saint Philips College*
Paula Looney, *Saint Philips College*
Fred Bakenhus, *Saint Philips College*
Lydia Casas, *Saint Philips College*
Gloria Guerra, *Saint Philips College*
Sounny Slitine, *Saint Philips College*
Jessica Lopez, *Saint Philips College*
Lorraine Lopez, *San Antonio College*
Peter Georgakis, *Santa Barbara City College*
Sandi Nieto-Navarro, *Santa Rosa Junior College*
Steve Drucker, *Santa Rosa Junior College*
Jean-Marie Magnier, *Springfield Tech Community College*
Dave Delrossi, *Tallahassee Community College*
Natalie Johnson, *Tarrant County College South*
Marilyn Peacock, *Tidewater Community College*
Yvonne Aucoin, *Tidewater Community College*
Cynthia Harris, *Triton College*
Jennifer Burkett, *Triton College*
Christyn Senese, *Triton College*
Jennifer Dale, *Triton College*
Patricia Hussey, *Triton College*
Glenn Jablonski, *Triton College*
Myrna La Rosa, *Triton College*
Michael Maltenfort, *Truman College*
Abdallah Shuaibi, *Truman College*
Marta Hidegkuti, *Truman College*
Sandra Wilder, *University of Akron*
Sandra Jovicic, *University of Akron*
Edward Migliore, *University of California–Santa Cruz*
Kelly Kohlmetz, *University of Wisconsin—Milwaukee*
Leah Rineck, *University of Wisconsin—Milwaukee*
Carolann Van Galder, *University of Wisconsin—Rock County*
Claudia Martinez, *Valencia College*
Stephen Toner, *Victor Valley Community College*
David Cooper, *Wake Tech Community College*
Karlata Elliott, *Wake Tech Community College*
Laura Kalbaugh, *Wake Tech Community College*
Kelly Vetter, *Wake Tech Community College*
Jacqui Fields, *Wake Tech Community College*
Jennifer Smeal, *Wake Tech Community College*
Shannon Vinson, *Wake Tech Community College*
Kim Walaski, *Wake Tech Community College*
Lisa Rombes, *Washtenaw Community College*
Maziar Ouliaeinia, *Western Iowa Tech Community College*
Keith McCoy, *Wilbur Wright College*

Mc Graw Hill ALEKS®

Create More Lightbulb Moments.

Every student has different needs and enters your course with varied levels of preparation. ALEKS® pinpoints what students already know, what they don't and, most importantly, what they're ready to learn next. Optimize your class engagement by aligning your course objectives to ALEKS® topics and layer on our textbook as an additional resource for students.

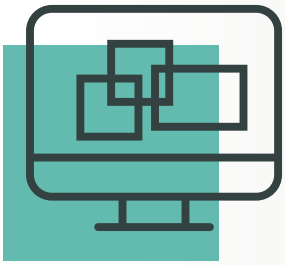
## ALEKS® Creates a Personalized and Dynamic Learning Path

ALEKS® creates an optimized path with an ongoing cycle of learning and assessment, celebrating students' small wins along the way with positive real-time feedback. Rooted in research and analytics, ALEKS® improves student outcomes by fostering better preparation, increased motivation, and knowledge retention.

*visit **bit.ly/whatmakesALEKSunique** to learn more about the science behind the most powerful adaptive learning tool in education!


## Preparation & Retention

The more prepared your students are, the more effective your instruction is. Because ALEKS® understands the prerequisite skills necessary for mastery, students are better prepared when a topic is presented to them. ALEKS® provides personalized practice and guides students to what they need to learn next to achieve mastery. ALEKS® improves knowledge and student retention through periodic knowledge checks and personalized learning paths. This cycle of learning and assessment ensures that students remember topics they have learned, are better prepared for exams, and are ready to learn new content as they continue into their next course.



## Flexible Implementation: Your Class Your Way!

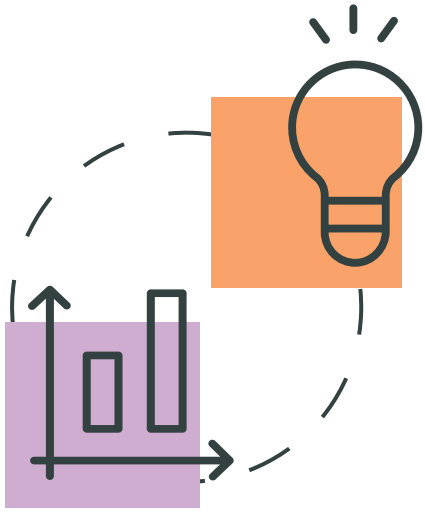
ALEKS® enables you to structure your course regardless of your instruction style and format. From a traditional classroom, to various co-requisite models, to an online prep course before the start of the term, ALEKS® can supplement your instruction or play a lead role in delivering the content.

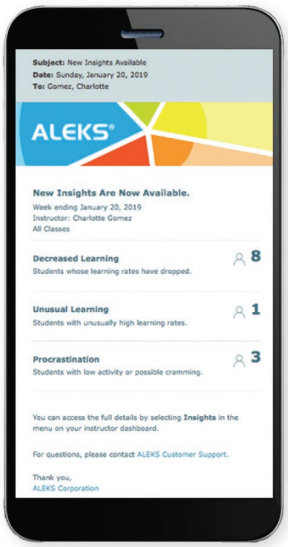
*visit **bit.ly/ALEKScasestudies** to see how your peers are delivering better outcomes across various course models!

## Outcomes & Efficacy

Our commitment to improve student outcomes services a wide variety of implementation models and best practices, from lecture-based to labs and co-reqs to summer prep courses. Our case studies illustrate our commitment to help you reach your course goals, and our research demonstrates our drive to support all students, regardless of their math background and preparation level.

*visit **bit.ly/outcomesandefficacy** to review empirical data from ALEKS® users around the country






## Turn Data Into Actionable Insights

ALEKS® Reports are designed to inform your instruction and create more meaningful interactions with your students when they need it the most. ALEKS® Insights alert you when students might be at risk of falling behind so that you can take immediate action. Insights summarize students exhibiting at least one of four negative behaviors that may require intervention, including Failed Topics, Decreased Learning, Unusual Learning, and Procrastination & Cramming.

DIGITAL EDGE50 AWARDS 2019

**Winner of 2019 Digital Edge 50 Award for Data Analytics!**

**bit.ly/ALEKS_MHE**

# Review of Basic Algebraic Concepts

# R

## CHAPTER OUTLINE



### *Mathematics and Consistency*

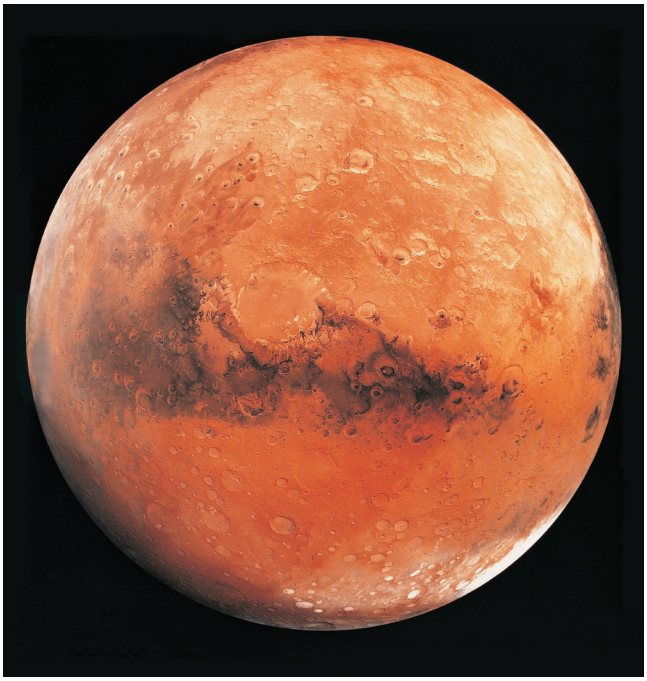
Many of the activities we perform every day follow a natural order. For example, we would not put on our shoes before putting on our socks, nor would a doctor begin surgery before giving an anesthetic.

In mathematics, it is also necessary to follow a prescribed order of operations to simplify an algebraic expression. This is important, for example, because we would not want two different engineers working on a space probe to Mars to interpret a mathematical statement differently.

Suppose that the high temperature for a summer day near the equator of Mars is 20°C. To convert this to degrees Fahrenheit $F$, we would substitute 20 for $C$ in the equation.

$$F = \frac{9}{5}C + 32 \xrightarrow{\text{Substitute 20 for } C} F = \frac{9}{5}(20) + 32$$

In this expression, the operation between $\frac{9}{5}$ and 20 is implied multiplication, and it is universally understood that multiplication is performed before addition. Thus,



Digital Vision/Getty Images

$$F = \frac{9}{5}(20) + 32 = 36 + 32 = 68. \qquad \text{The temperature in Fahrenheit is } 68°\text{F}.$$

If an engineer had erroneously added 20 and 32 first and then multiplied by $\frac{9}{5}$, a different temperature of 93.6°F would result. This illustrates the importance of a prescribed order for mathematical operations.

# Section R.1 Sets of Numbers and Interval Notation

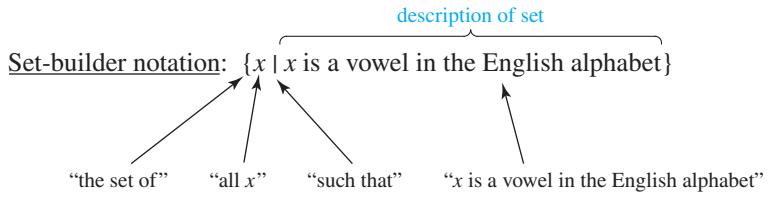
## Concepts



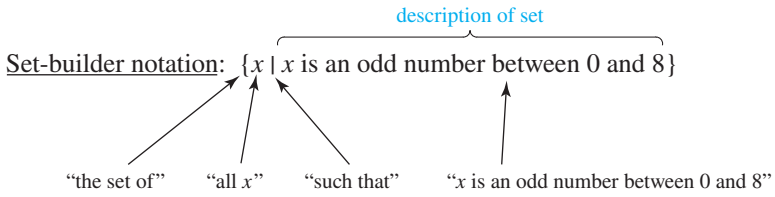
## 1. The Set of Real Numbers

Algebra is a powerful mathematical tool that is used to solve real-world problems in science, business, and many other fields. We begin our study of algebra with a review of basic definitions and notations used to express algebraic relationships.

In mathematics, a collection of items (called elements) is called a **set**, and the set braces { } are used to enclose the elements of the set. For example, the set {a, e, i, o, u} represents the vowels in the English alphabet. The set {1, 3, 5, 7} represents the first four positive odd numbers. Another method to express a set is to *describe* the elements of the set by using **set-builder notation**. Consider the set {a, e, i, o, u} in set-builder notation.




Consider the set {1, 3, 5, 7} in set-builder notation.




Several sets of numbers are used extensively in algebra. The numbers you are familiar with in day-to-day calculations are elements of the set of **real numbers**. These numbers can be represented graphically on a horizontal number line with a point labeled as 0. Positive real numbers are graphed to the right of 0, and negative real numbers are graphed to the left. Each point on the number line corresponds to exactly one real number, and for this reason, the line is called the **real number line** (Figure R-1).


**Figure R-1**

Several sets of numbers are **subsets** (or part) of the set of real numbers. These are

The set of natural numbers

The set of whole numbers

The set of integers

The set of rational numbers

The set of irrational numbers

### Natural Numbers, Whole Numbers, and Integers

The set of **natural numbers** is {1, 2, 3, . . .}.
The set of **whole numbers** is {0, 1, 2, 3, . . .}.
The set of **integers** is {. . . , −3, −2, −1, 0, 1, 2, 3, . . .}.

The set of rational numbers consists of all the numbers that can be defined as a ratio of two integers.

### Rational Numbers

The set of **rational numbers** is $\left\{\frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \text{ does not equal zero}\right\}$.

### Example 1 Identifying Rational Numbers

Show that each number is a rational number by finding two integers whose ratio equals the given number.

**a.** $\frac{-4}{7}$ **b.** 8 **c.** $0.\overline{6}$ **d.** 0.87

**Solution:**

**a.** $\frac{-4}{7}$ is a rational number because it can be expressed as the ratio of the integers $-4$ and 7.

**b.** 8 is a rational number because it can be expressed as the ratio of the integers 8 and 1 $\left(8 = \frac{8}{1}\right)$. In this example we see that *an integer is also a rational number.*

**c.** $0.\overline{6}$ represents the repeating decimal 0.6666666 . . . and can be expressed as the ratio of 2 and 3 $\left(0.\overline{6} = \frac{2}{3}\right)$. In this example we see that *a repeating decimal is a rational number.*

**d.** 0.87 is the ratio of 87 and 100 $\left(0.87 = \frac{87}{100}\right)$. In this example we see that *a terminating decimal is a rational number.*

**Skill Practice** Show that the numbers are rational by writing them as a ratio of integers.

**1.** $\frac{-9}{8}$ **2.** 0 **3.** $0.\overline{3}$ **4.** 0.45

**TIP:** Any rational number can be represented by a terminating decimal or by a repeating decimal.

Some real numbers such as the number $\pi$ (pi) cannot be represented by the ratio of two integers. In decimal form, an irrational number is a nonterminating, nonrepeating decimal. The value of $\pi$, for example, can be approximated as $\pi \approx 3.1415926535897932$. However, the decimal digits continue indefinitely with no pattern. Other examples of irrational numbers are the square roots of nonperfect squares, such as $\sqrt{3}$ and $\sqrt{10}$.

### Irrational Numbers

The set of **irrational numbers** is a subset of the real numbers whose elements cannot be written as a ratio of two integers.

*Note:* An irrational number cannot be written as a terminating decimal or as a repeating decimal.

The set of real numbers consists of both the rational numbers and the irrational numbers. The relationships among the sets of numbers discussed thus far are illustrated in Figure R-2.

**Answers**

**1.** $\frac{-9}{8}$ **2.** $\frac{0}{1}$
**3.** $\frac{1}{3}$ **4.** $\frac{45}{100}$ or $\frac{9}{20}$

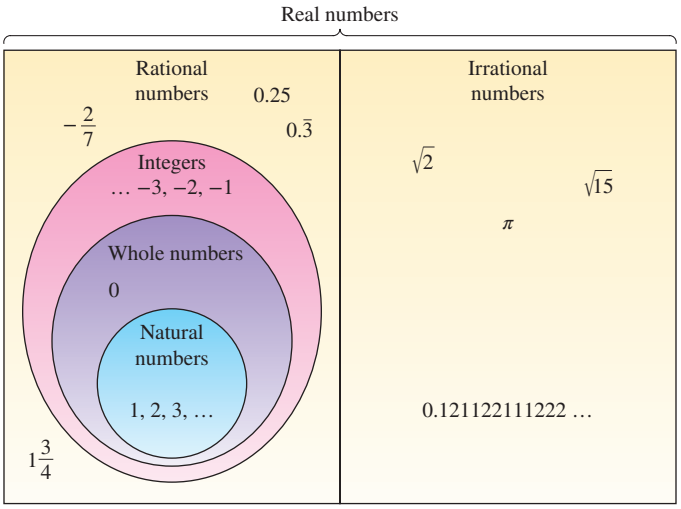



**Figure R-2**

## Example 2 Classifying Numbers by Set

Check the set(s) to which each number belongs. The numbers may belong to more than one set.

| | Natural Numbers | Whole Numbers | Integers | Rational Numbers | Irrational Numbers | Real Numbers |
|---|---|---|---|---|---|---|
| $-6$ | | | | | | |
| $\sqrt{23}$ | | | | | | |
| $-\frac{2}{7}$ | | | | | | |
| 3 | | | | | | |
| $2.\overline{3}$ | | | | | | |

**Solution:**

| | Natural Numbers | Whole Numbers | Integers | Rational Numbers | Irrational Numbers | Real Numbers |
|---|---|---|---|---|---|---|
| $-6$ | | | ✓ | ✓ | | ✓ |
| $\sqrt{23}$ | | | | | ✓ | ✓ |
| $-\frac{2}{7}$ | | | | ✓ | | ✓ |
| 3 | ✓ | ✓ | ✓ | ✓ | | ✓ |
| $2.\overline{3}$ | | | | ✓ | | ✓ |

**Skill Practice**

**5.** Check the set(s) to which each number belongs.

| | 1 | 0.47 | $\sqrt{5}$ | $-\frac{1}{2}$ |
|---|---|---|---|---|
| **Natural** | | | | |
| **Whole** | | | | |
| **Integer** | | | | |
| **Rational** | | | | |
| **Irrational** | | | | |
| **Real** | | | | |

**Answer**

**5.**

| | 1 | 0.47 | $\sqrt{5}$ | $-\frac{1}{2}$ |
|---|---|---|---|---|
| Natural | ✓ | | | |
| Whole | ✓ | | | |
| Integer | ✓ | | | |
| Rational | ✓ | ✓ | | ✓ |
| Irrational | | | ✓ | |
| Real | ✓ | ✓ | ✓ | ✓ |

## 2. Inequalities

The relative value of two numbers can be compared by using the real number line. We say that $a$ is less than $b$ (written mathematically as $a < b$) if $a$ lies to the left of $b$ on the number line.


$a < b$

We say that $a$ is greater than $b$ (written mathematically as $a > b$) if $a$ lies to the right of $b$ on the number line.


$a > b$

From looking at the number line, note that $a > b$ is the same as $b < a$. Table R-1 summarizes the relational operators that compare two real numbers $a$ and $b$.

**Table R-1**

| Mathematical Expression | Translation | Other Meanings |
|---|---|---|
| $a < b$ | $a$ is less than $b$ | $b$ exceeds $a$<br>$b$ is greater than $a$ |
| $a > b$ | $a$ is greater than $b$ | $a$ exceeds $b$<br>$b$ is less than $a$ |
| $a \le b$ | $a$ is less than or equal to $b$ | $a$ is at most $b$<br>$a$ is no more than $b$ |
| $a \ge b$ | $a$ is greater than or equal to $b$ | $a$ is no less than $b$<br>$a$ is at least $b$ |
| $a = b$ | $a$ is equal to $b$ | |
| $a \ne b$ | $a$ is not equal to $b$ | |
| $a \approx b$ | $a$ is approximately equal to $b$ | |

The symbols $<$, $>$, $\le$, $\ge$, and $\ne$ are called inequality signs, and the expressions $a < b$, $a > b$, $a \le b$, $a \ge b$, and $a \ne b$ are called **inequalities**.

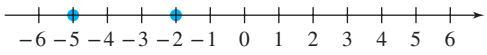
**Example 3** **Ordering Real Numbers**

Fill in the blank with the appropriate inequality sign: $<$ or $>$

**a.** $-2$ _____ $-5$ **b.** $\frac{4}{7}$ _____ $\frac{3}{5}$ **c.** $-1.3$ _____ $-1.\overline{3}$

**Solution:**

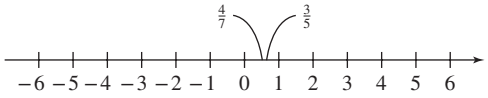
**a.** $-2$ ___>___ $-5$



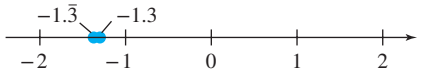

**b.** To compare $\frac{4}{7}$ and $\frac{3}{5}$, write the fractions as equivalent fractions with a common denominator.

$\frac{4}{7} \cdot \frac{5}{5} = \frac{20}{35}$ and $\frac{3}{5} \cdot \frac{7}{7} = \frac{21}{35}$

Because $\frac{20}{35} < \frac{21}{35}$, then $\frac{4}{7}$ ___<___ $\frac{3}{5}$



**c.** $-1.3$ ___≥___ $-1.33333\ldots$

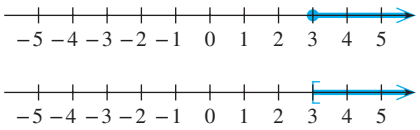



**Skill Practice** Fill in the blanks with the appropriate sign, $<$ or $>$.

**6.** $2$ _____ $-12$ **7.** $\frac{1}{4}$ _____ $\frac{2}{9}$ **8.** $-7.\overline{2}$ _____ $-7.2$

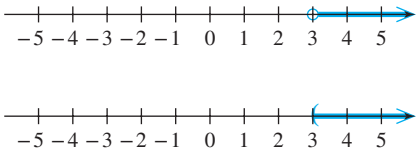
## 3. Interval Notation

The set $\{x \mid x \geq 3\}$ represents all real numbers greater than or equal to 3. This set can be illustrated graphically on the number line.



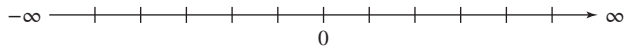

By convention, a closed circle ● or a square bracket [ is used to indicate that an "endpoint" ($x = 3$) *is included* in the set.

The set $\{x \mid x > 3\}$ represents all real numbers strictly greater than 3. This set can be illustrated graphically on the number line.




By convention, an open circle ○ or a parenthesis ( is used to indicate that an "endpoint" ($x = 3$) is *not* included in the set.

Notice that the sets $\{x \mid x \geq 3\}$ and $\{x \mid x > 3\}$ consist of an infinite number of elements that cannot all be listed. Another method to represent the elements of such sets is by using **interval notation**. To understand interval notation, first consider the real number line, which extends infinitely far to the left and right. The symbol $\infty$ is used to represent infinity. The symbol $-\infty$ is used to represent negative infinity.




To express a set of real numbers in interval notation, sketch the graph first, using the symbols ( ) or [ ]. Then use these symbols at the endpoints to define the interval.

**Example 4** **Expressing Sets by Using Interval Notation**

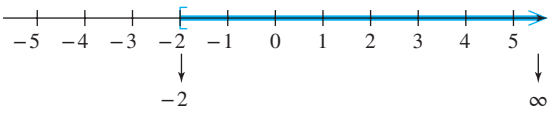
Graph each set on the number line, and express the set in interval notation.

**a.** $\{x \mid x \geq -2\}$ **b.** $\{p \mid p > -2\}$

**Solution:**

**a.** Set-builder notation: $\{x \mid x \geq -2\}$

Graph:




Interval notation: $[-2, \infty)$

**FOR REVIEW**

Recall that an inequality may be written with the variable on either side of the inequality sign. The statement $x \geq -2$ is equivalent to $-2 \leq x$.

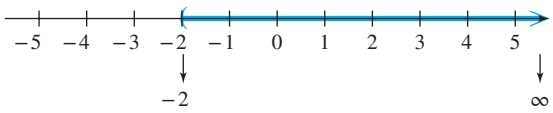
**Answers**

**6.** $>$ **7.** $>$ **8.** $<$

The graph of the set $\{x \mid x \geq -2\}$ "begins" at $-2$ and extends infinitely far to the right. The corresponding interval notation "begins" at $-2$ and extends to $\infty$. Notice that a square bracket [ is used at $-2$ for both the graph and the interval notation to include $x = -2$. *A parenthesis is always used at $\infty$ and at $-\infty$ because there is no endpoint.*

**b.** Set-builder notation: $\{p \mid p > -2\}$

Graph:




Interval notation: $(-2, \infty)$

**Skill Practice** Graph each set, and express the set in interval notation.

**9.** $\{w \mid w \geq -7\}$ **10.** $\{x \mid x < 0\}$

In general, we use the following guidelines when applying interval notation.

## Using Interval Notation

- The endpoints used in interval notation are always written from left to right. That is, the smaller number is written first, followed by a comma, followed by the larger number.
- Parentheses ) or ( indicate that an endpoint is *excluded* from the set.
- Square brackets ] or [ indicate that an endpoint is *included* in the set.
- Parentheses are always used with $\infty$ or $-\infty$.

## Example 5 Expressing Sets by Using Interval Notation

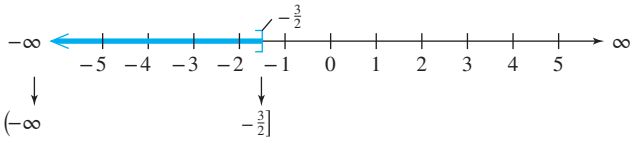
Graph each set on the number line, and express the set in interval notation.

**a.** $\left\{z \mid z \leq -\frac{3}{2}\right\}$ **b.** $\{x \mid -4 < x \leq 2\}$

**Solution:**

**a.** Set-builder notation: $\left\{z \mid z \leq -\frac{3}{2}\right\}$

Graph:




Interval notation: $\left(-\infty, -\frac{3}{2}\right]$

The graph of the set $\left\{z \mid z \leq -\frac{3}{2}\right\}$ extends infinitely far to the left. Interval notation is always written from left to right. Therefore, $-\infty$ is written first, followed by a comma, and then followed by the right-hand endpoint $-\frac{3}{2}$.

**b.** The inequality $-4 < x \leq 2$ means that $x$ is greater than $-4$ and also less than or equal to 2. More concisely, we can say that $x$ represents the real numbers *between* $-4$ and 2, including the endpoint, 2.

Set-builder notation: $\{x \mid -4 < x \leq 2\}$

Graph:


Interval notation: $(-4, 2]$

**Answers**

**9.**


$[-7, \infty)$

**10.**

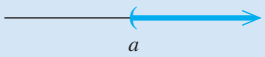
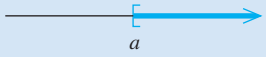
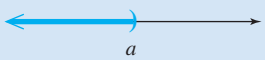
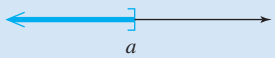
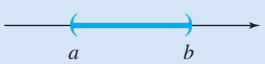
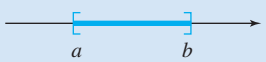
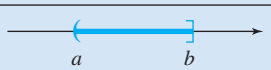
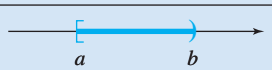

$(-\infty, 0)$

**Skill Practice** Graph the set on the number line, and express the set in interval notation.

**11.** $\{w \mid w \geq -\frac{5}{3}\}$ **12.** $\{y \mid -7 \leq y < 4\}$

Table R-2 summarizes interval notation.

**Table R-2**

| Interval Notation | Graph | Interval Notation | Graph |
|---|---|---|---|
| $(a, \infty)$ |  | $[a, \infty)$ |  |
| $(-\infty, a)$ |  | $(-\infty, a]$ |  |
| $(a, b)$ |  | $[a, b]$ |  |
| $(a, b]$ |  | $[a, b)$ |  |

## 4. Translations Involving Inequalities

In Table R-1, we learned that phrases such as *at least*, *at most*, *no more than*, *no less than*, and *between* can be translated into mathematical terms by using inequality signs.

**Example 6** Translating Inequalities

The intensity of a hurricane is often defined according to its maximum sustained winds, for which wind speed is measured to the nearest mile per hour. Translate the italicized phrases into mathematical inequalities.

**a.** A tropical storm is updated to hurricane status if the sustained wind speed, $w$, *is at least 74 mph*.

**b.** Hurricanes are categorized according to intensity by the Saffir-Simpson scale. On a scale of 1 to 5, a category 5 hurricane is the most destructive. A category 5 hurricane has sustained winds, $w$, *exceeding 155 mph*.

**c.** A category 4 hurricane has sustained winds, $w$, *of at least 131 mph but no more than 155 mph*.

**Solution:**

**a.** $w \geq 74$ mph **b.** $w > 155$ mph **c.** $131 \text{ mph} \leq w \leq 155 \text{ mph}$

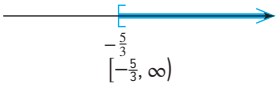
**Skill Practice** Translate the italicized phrase to a mathematical inequality.

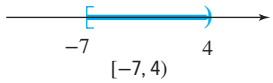
**13.** The gas mileage, $m$, for an economy car is *at least 30 mpg*.

**14.** The gas mileage, $m$, for a motorcycle is *more than 45 mpg*.

**15.** The gas mileage, $m$, for an SUV is *at least 10 mpg, but no more than 20 mpg*.

**Answers**

**11.** 


**12.** 


**13.** $m \geq 30$

**14.** $m > 45$

**15.** $10 \leq m \leq 20$

## Section R.1 Activity

For Exercises A.1–A.6, refer to set $A$. $A = \left\{-5, -2.\overline{5}, \sqrt{7}, 0, \frac{7}{4}, 4, -2.5\right\}$

**A.1.** Which elements from $A$ are natural numbers?

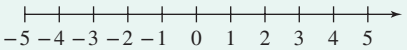
**A.2.** Which elements from $A$ are whole numbers?

**A.3.** Which elements from $A$ are integers?

**A.4.** Which elements from $A$ are rational numbers?

**A.5.** Which elements from $A$ are irrational numbers?

**A.6.** Plot the elements from $A$ on the number line.




Inequality statements surround us in day-to-day life. In Exercises A.7–A.9, let $x$ represent the unknown quantity, and write a mathematical inequality to represent the given statement.

**A.7.** A child must be at least 44 inches tall to ride Space Mountain.

**A.8.** According to the posted speed limit on a country road, a driver traveling at most 35 mph will not get a speeding ticket.

**A.9.** To preregister to vote in the United States, a person must be at least 16, but less than 18 years old.

**A.10.** **a.** Graph the inequality $x < 1$. ⟶

**b.** Graph the inequality $x \leq 1$. ⟶

**c.** Explain when to use parentheses, ( or ), versus brackets [ or ], when graphing an inequality.

**d.** Write the inequality $x < 1$ in interval notation.

**e.** Write the inequality $x \leq 1$ in interval notation.

**f.** Explain when to use parentheses, ( or ), versus brackets [ or ], when using interval notation.

For Exercises A.11–A.12, write the set in interval notation.

**A.11.** $\{x | -4 < x \leq -1\}$

**A.12.** $\left\{x \middle| \frac{3}{2} \leq x\right\}$

## Practice Exercises Section R.1

### Study Skills Exercises

Mindset plays an important role in your approach to learning mathematics. Mindset consists of our thoughts, beliefs, and attitudes about our abilities based on lifetime experiences. There are two types of mindsets: fixed mindsets and growth mindsets. People with a fixed mindset believe that they are born with a certain amount of intelligence that cannot be changed despite their actions. On the other hand, a person with a growth mindset believes that intelligence is dynamic and can be increased with effort and learning. What type of mindset do you have? Think about the following questions:

- Have you said to yourself, "I'm just not good at math"?
- Do you believe you lack the necessary skills to understand math?
- Can you recall an experience that has positively impacted your self-confidence in mathematics?