

 Cengage

Precalculus

Mathematics for Calculus

Eighth Edition

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Exponents and Radicals

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x}$$

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x} \sqrt[n]{y}$$

$$\sqrt[n]{\sqrt[m]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[nm]{x}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

Special Products

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

Factoring Formulas

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Quadratic Formula

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$.

If $a < b$, then $a + c < b + c$.

If $a < b$ and $c > 0$, then $ca < cb$.

If $a < b$ and $c < 0$, then $ca > cb$.

If $a > 0$, then

$$|x| = a \text{ means } x = a \text{ or } x = -a.$$

$$|x| < a \text{ means } -a < x < a.$$

$$|x| > a \text{ means } x > a \text{ or } x < -a.$$

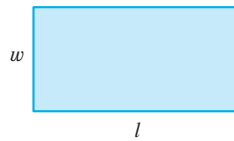
Geometric Formulas

Formulas for area A , perimeter P , circumference C , and volume V :

Rectangle

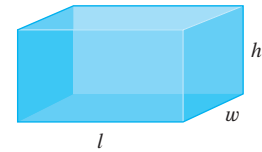
$$A = lw$$

$$P = 2l + 2w$$



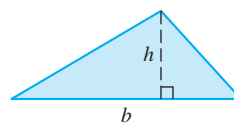
Box

$$V = lwh$$



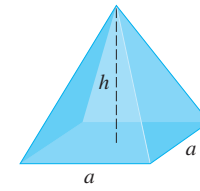
Triangle

$$A = \frac{1}{2}bh$$



Pyramid

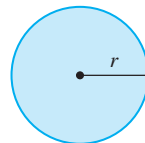
$$V = \frac{1}{3}ha^2$$



Circle

$$A = \pi r^2$$

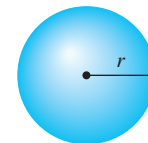
$$C = 2\pi r$$



Sphere

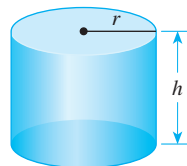
$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$



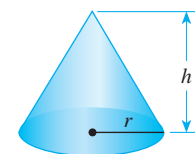
Cylinder

$$V = \pi r^2 h$$



Cone

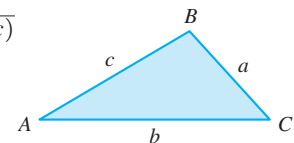
$$V = \frac{1}{3}\pi r^2 h$$



Heron's Formula

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$



Distance and Midpoint Formulas

Distance between $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of P_1P_2 : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Lines

Slope of line through $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-slope equation of line through $P_1(x_1, y_1)$ with slope m

$$y - y_1 = m(x - x_1)$$

Slope-intercept equation of line with slope m and y-intercept b

$$y = mx + b$$

Two-intercept equation of line with x-intercept a and y-intercept b

$$\frac{x}{a} + \frac{y}{b} = 1$$

Logarithms

$y = \log_a x$ means $a^y = x$

$$\log_a a^x = x$$

$$a^{\log_a x} = x$$

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log x = \log_{10} x$$

$$\ln x = \log_e x$$

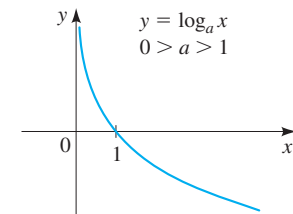
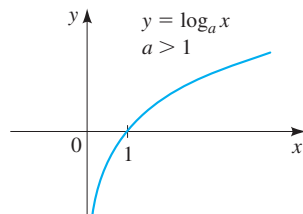
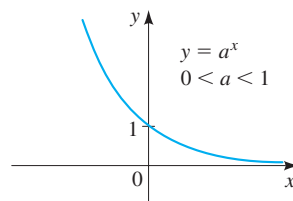
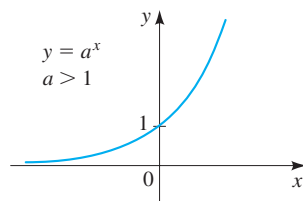
$$\log_a xy = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^b = b \log_a x$$

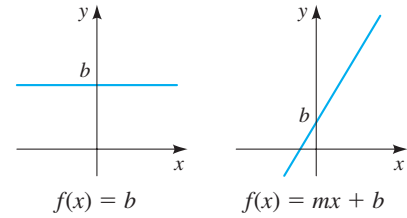
$$\log_b x = \frac{\log_a x}{\log_a b}$$

Exponential and Logarithmic Functions

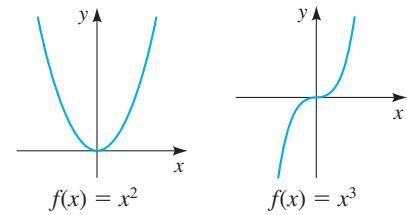


Graphs of Functions

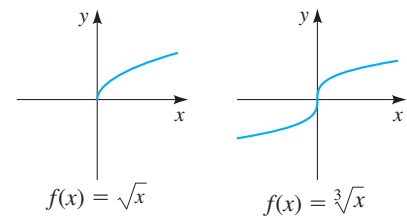
Linear functions: $f(x) = mx + b$



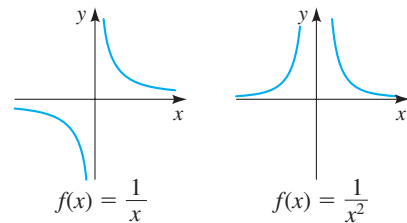
Power functions: $f(x) = x^n$



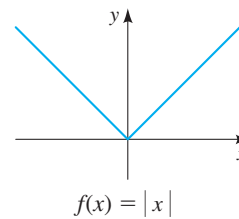
Root functions: $f(x) = \sqrt[n]{x}$



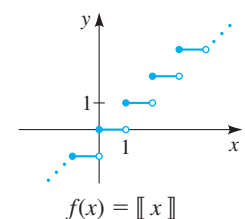
Reciprocal functions: $f(x) = 1/x^n$



Absolute value function



Greatest integer function



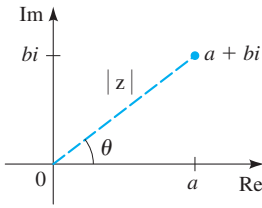
Complex Numbers

For the complex number $z = a + bi$

the **conjugate** is $\bar{z} = a - bi$

the **modulus** is $|z| = \sqrt{a^2 + b^2}$

the **argument** is θ , where $\tan \theta = b/a$



Polar form of a complex number

For $z = a + bi$, the **polar form** is

$$z = r(\cos \theta + i \sin \theta)$$

where $r = |z|$ is the modulus of z and θ is the argument of z

De Moivre's Theorem

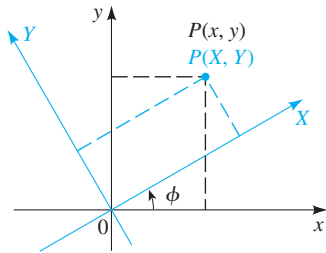
$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$$

$$\sqrt[n]{z} = [r(\cos \theta + i \sin \theta)]^{1/n}$$

$$= r^{1/n} \left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n} \right)$$

where $k = 0, 1, 2, \dots, n - 1$

Rotation of Axes



Rotation of axes formulas

$$x = X \cos \phi - Y \sin \phi$$

$$y = X \sin \phi + Y \cos \phi$$

Angle-of-rotation formula for conic sections

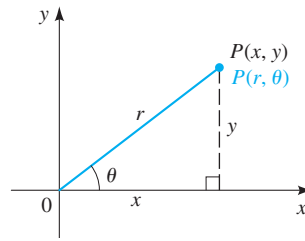
To eliminate the xy -term in the equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

rotate the axis by the angle ϕ that satisfies

$$\cot 2\phi = \frac{A - C}{B}$$

Polar Coordinates



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

Sums of Powers of Integers

$$\sum_{k=1}^n 1 = n$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$$

The Derivative

The **average rate of change** of f between a and b is

$$\frac{f(b) - f(a)}{b - a}$$

The **derivative** of f at a is

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

Area Under the Graph of f

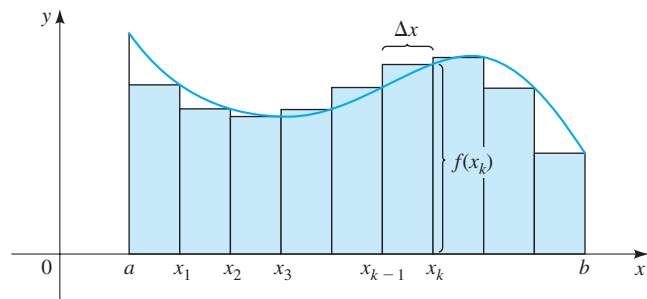
The **area under the graph of f** on the interval $[a, b]$ is the limit of the sum of the areas of approximating rectangles

$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$$

where

$$\Delta x = \frac{b - a}{n}$$

$$x_k = a + k \Delta x$$



Sequences and Series

Arithmetic

$$a, a + d, a + 2d, a + 3d, \dots \quad \text{or} \quad a_n = a + (n - 1)d$$

$$S_n = \sum_{k=1}^n a_k = \frac{n}{2}[2a + (n - 1)d] = n\left(\frac{a + a_n}{2}\right)$$

Geometric

$$a, ar, ar^2, ar^3, \dots \quad \text{or} \quad a_n = ar^{n-1}$$

$$S_n = \sum_{k=1}^n a_k = a \frac{1 - r^n}{1 - r}$$

Infinite geometric series

$$\sum_{k=1}^{\infty} ar^{k-1} = a + ar + ar^2 + ar^3 + \dots$$

If $|r| < 1$ the series converges and its sum is

$$S = \frac{a}{1 - r}$$

If $|r| \geq 1$ the series diverges.

The Binomial Theorem

Binomial Theorem

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

Binomial coefficients

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (r \leq n), \quad \text{where } n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$$

Finance

Compound interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A is the amount after t years, P is the principal, r is the interest rate, and the interest is compounded n times per year.

Continuously compounded interest

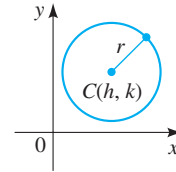
$$A = Pe^{rt}$$

where A is the amount after t years, P is the principal, r is the interest rate, and the interest is compounded continuously.

Conic Sections

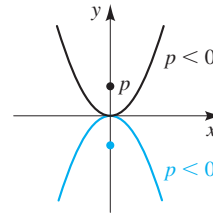
Circles

$$(x - h)^2 + (y - k)^2 = r^2$$



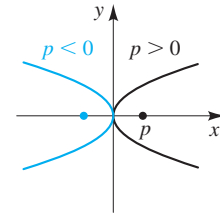
Parabolas

$$x^2 = 4py$$

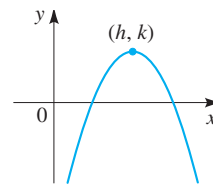


Focus $(0, p)$, directrix $y = -p$

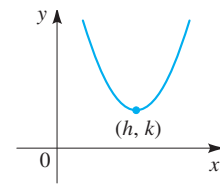
$$y^2 = 4px$$



Focus $(p, 0)$, directrix $x = -p$



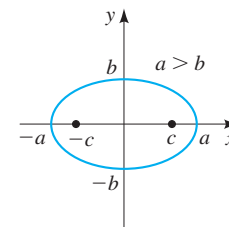
$$y = a(x - h)^2 + k, \quad a < 0, \quad h > 0, \quad k > 0$$



$$y = a(x - h)^2 + k, \quad a > 0, \quad h > 0, \quad k > 0$$

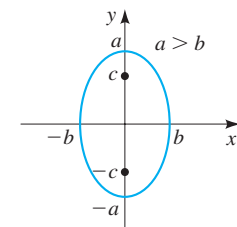
Ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

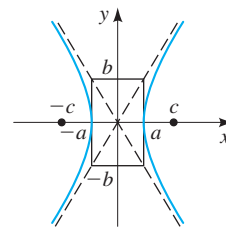
$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

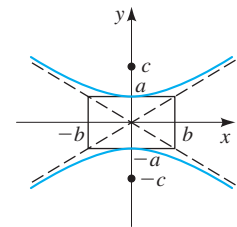
Hyperbolas

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$
Asymptotes: $y = \pm \frac{b}{a}x$

$$-\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$
Asymptotes: $y = \pm \frac{a}{b}x$

Angle Measurement

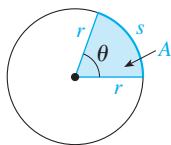
$$\pi \text{ radians} = 180^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad} \quad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$s = r\theta \quad A = \frac{1}{2}r^2\theta \quad (\theta \text{ in radians})$$

To convert from degrees to radians, multiply by $\frac{\pi}{180}$.

To convert from radians to degrees, multiply by $\frac{180}{\pi}$.



Trigonometric Functions of Real Numbers

$$\sin t = y$$

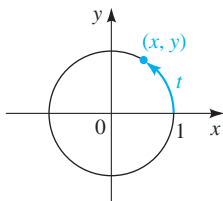
$$\csc t = \frac{1}{y}$$

$$\cos t = x$$

$$\sec t = \frac{1}{x}$$

$$\tan t = \frac{y}{x}$$

$$\cot t = \frac{x}{y}$$



Trigonometric Functions of Angles

$$\sin \theta = \frac{y}{r}$$

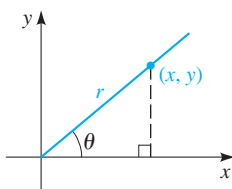
$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

$$\cot \theta = \frac{x}{y}$$



Right Angle Trigonometry

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

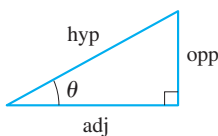
$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

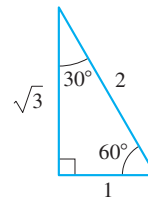
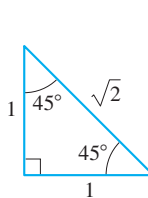
$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$



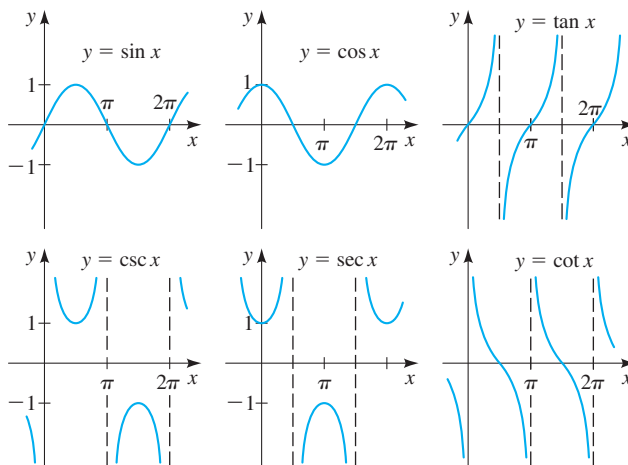
Special Values of the Trigonometric Functions

| θ | radians | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ |
|-------------|----------|---------------|---------------|---------------|
| 0° | 0 | 0 | 1 | 0 |
| 30° | $\pi/6$ | $1/2$ | $\sqrt{3}/2$ | $\sqrt{3}/3$ |
| 45° | $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| 60° | $\pi/3$ | $\sqrt{3}/2$ | $1/2$ | $\sqrt{3}$ |
| 90° | $\pi/2$ | 1 | 0 | — |
| 180° | π | 0 | -1 | 0 |
| 270° | $3\pi/2$ | -1 | 0 | — |

Special Triangles



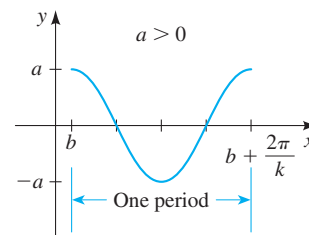
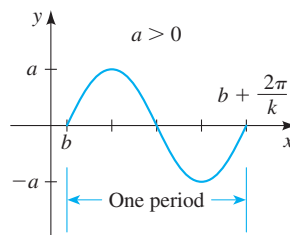
Graphs of the Trigonometric Functions



Sine and Cosine Curves

$$y = a \sin k(x - b) \quad (k > 0)$$

$$y = a \cos k(x - b) \quad (k > 0)$$



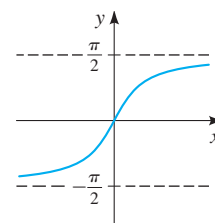
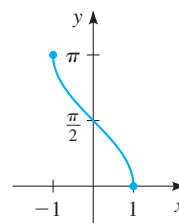
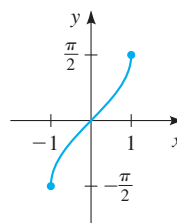
amplitude: $|a|$ period: $2\pi/k$ horizontal shift: b

Graphs of the Inverse Trigonometric Functions

$$y = \sin^{-1}x$$

$$y = \cos^{-1}x$$

$$y = \tan^{-1}x$$



Fundamental Identities

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\sin^2 x + \cos^2 x = 1 \quad 1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Reduction Identities

$$\sin(x + \pi) = -\sin x$$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\cos(x + \pi) = -\cos x$$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$\tan(x + \pi) = \tan x$$

$$\tan\left(x + \frac{\pi}{2}\right) = -\cot x$$

Addition and Subtraction Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \quad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2 \cos^2 x - 1$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$= 1 - 2 \sin^2 x$$

Formulas for Reducing Powers

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Half-Angle Formulas

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

Product-to-Sum and Sum-to-Product Identities

$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$$

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

The Laws of Sines and Cosines

The Law of Sines

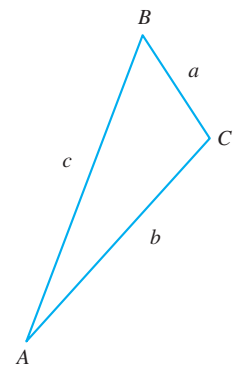
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Precalculus

Mathematics for Calculus

Eighth Edition

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About the Cover

The cover photograph shows a small portion of Kö-Bogen (the King's Bow), a large-scale office and retail complex in Düsseldorf, Germany. The complex was designed by the world-renowned architect Daniel Libeskind. The walls of the buildings feature long sinuous curves. The design and construction of such curves requires precise geometric descriptions of the curves;

mathematical formulas are used for computing the structural stability and hence the feasibility of constructing such large curves in steel and concrete. In this book we explore geometric descriptions of certain curves that can help in constructing them at any scale. (See *Focus on Modeling, Conics in Architecture*, following Chapter 10.)

Precalculus

Mathematics for Calculus

Eighth Edition

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With the assistance of Phyllis Panman



Precalculus: Mathematics for Calculus,
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Preface

The art of teaching is the art of assisting discovery.

Mark Van Doren



What do students really need to know to be prepared for calculus? What tools do instructors really need to assist their students in preparing for calculus? These two questions have motivated the writing of this book.

To be prepared for calculus a student needs not only technical skill but also a clear understanding of concepts. Indeed, *conceptual understanding* and *technical skill* go hand in hand, each reinforcing the other. A student also needs to gain experience in *problem-solving*, as well as an appreciation for the power and utility of mathematics in *modeling* the real world. Every feature of this textbook is devoted to fostering these goals.

In this Eighth Edition our objective is to further enhance the effectiveness of the book as a teaching tool for instructors and as a learning tool for students. Many of the changes in this new edition are a result of suggestions we have received from instructors and students who are using the current edition, recommendations from reviewers, insights we have gained from our own experience teaching from the text, and conversations we have had with colleagues and students. In all the changes, both small and large, we have retained the features that have contributed to the success of this book.

What's New in the Eighth Edition?

The overall structure of the book remains largely the same, but we have made many improvements. Some of the changes are as follows.

- **Exercises** More than 20% of the exercises are new, including new types of conceptual and skill exercises.
- **Emphasis on Problem-Solving** The Prologue *Principles of Problem Solving* contains several principles that can be used to tackle any problem. This edition includes new exercises designed to give students the opportunity to experience the process of problem-solving for themselves; each such exercise is identified by the icon  and includes a suggestion for which problem-solving principle to try. (See, for instance, Exercises 1.4.105, 1.9.116 and 118, 2.3.76, 2.6.103, 2.7.93, 6.1.71 and 74, 6.2.75, and 6.3.75–76.)
- **Special Application Exercises** In many sections there are new real-word application exercises that include some background information about the formulas or data used; we trust these exercises will capture students's interest. (See, for instance, Exercises 1.2.99, 1.8.127, 1.9.114, 1.12.53, 2.1.86, 2.5.48, 2.6.96, 3.7.62, 4.2.41, 4.5.96, and 4.6.29–31.)
- **Preparing for Calculus** Exercises that highlight skills which are needed in a calculus course are now identified by the icon . (See, for instance, Exercises 1.3.135–138, 2.4.19–24, 2.7.65–78, 4.2.25–28, 4.3.87–90, and 5.2.75–82.) The presentations, notation, features, and tone of this book are designed to be consistent with the Stewart Calculus textbooks.
- **Equations and Their Graphs** In this edition, we increase our emphasis on the essential relationship between an equation and its graph. (See, for instance, Example 1.11.1 and Exercise 2.R.83.) Where appropriate, the end-of-chapter review exercises now include a new exercise in which students are asked to

- match equations with their graphs and give reasons for their choices. (See, for instance, the Review Exercises 1.R.160, 2.R.111, 3.R.109, 4.R.111, and 5.R.75.)
- **Discovery Projects** Several new *Discovery Projects* have been added. Each project is briefly described in the appropriate section. [See, for instance, the projects on Weighing the Whole World (Section 1.5), Hours of Daylight (Section 5.6), Collision (Section 8.4), and Symmetry (Section 10.4).] The projects are available at the book companion website www.stewartmath.com.
 - **Expanded Problems in WebAssign** This edition's new online Expanded Problems further reinforce understanding by asking students to show the specific steps of their work or to explain the reasoning behind the answers they've provided.
 - **Geometry Review** Appendix A *Geometry Review* has been expanded to contain all the geometry background referred to in the textbook. The new topics include theorems on parallel lines and theorems on circles.
 - **Chapter 1 Fundamentals** In this chapter there are new groups of exercises with the title *Putting it All Together* that are intended to encourage students to first recognize the type of problem and then decide which method to use to solve it. (See, for instance, Exercises 1.2.77–80 and 1.5.103–118.)
 - **Chapter 2 Functions** This chapter now includes a subsection on relations, which is introduced in the context of answering the following question: Which tables of values (or relations) represent functions? This topic complements the preceding subsections Which Graphs Represent Functions? and Which Equations Represent Functions?
 - **Chapter 4 Exponential and Logarithmic Functions** This chapter now introduces the concept of logistic growth as a real-world application of exponential functions.
 - **Chapter 5 Trigonometric Functions: Unit Circle Approach** The material on graphing trigonometric functions has been restructured. The chapter now includes step-by-step guidelines for graphing transformations of trigonometric functions.
 - **Chapter 7 Analytic Trigonometry** The exercises on proving trigonometric identities have been restructured; these exercises now include a greater variety of trigonometric identities for students to try to prove.
 - **Chapter 8 Polar Equation, Parametric Equations, Vectors** This is a new chapter that combines the topics of polar coordinates and parametric equations, together with the topic of vectors in the plane (previously in Chapter 9).


Teaching with the Help of This Book

We are keenly aware that good teaching comes in many forms and that there are many effective approaches to teaching and learning the concepts and skills of precalculus. The organization and exposition of the topics in this book are designed to accommodate different teaching and learning styles. In particular, each topic is presented algebraically, graphically, numerically, and verbally, with emphasis on the relationships between these different representations.

The features in this book—*Focus on Modeling* sections, *Discovery Projects*, *Discuss/Discover/Prove/Write* exercises, historical insights, and *Mathematics in the Modern World* vignettes—provide a variety of enhancements to a central core of fundamental concepts and skills. Our aim is to provide instructors and their students with the tools they need to navigate their own course toward discovering precalculus mathematics.

The following are some special features that can be used to complement different teaching and learning approaches.

Exercise Sets The most important way to foster conceptual understanding and hone technical skill is through the problems that the instructor assigns. To that end we have provided a wide selection of exercises.

- **Concept Exercises** These exercises are at the beginning of each exercise set and ask students to state and use basic facts about the topics of each section.
- **Skills Exercises** These exercises reinforce and provide practice with all the learning objectives of each section. They comprise the core of each exercise set.
- **Skills Plus Exercises** The *Skills Plus* exercises contain challenging problems that often require the synthesis of previously learned material with new concepts.
- **Applications Exercises** We have included substantial applied problems from many different real-world contexts. We believe that these exercises will capture students' interest.
- **Discovery, Writing, and Group Learning** Each exercise set ends with a block of exercises labeled *Discuss* ■ *Discover* ■ *Prove* ■ *Write*. These exercises are designed to encourage students to experiment, preferably in groups, with the concepts developed in the section and then to write about what they have learned rather than simply looking for the answer. The *Prove* exercises highlight the importance of deriving a formula. (See for instance Exercises 2.6.97–103 and 2.8.108–112.)
- **Problem-Solving Exercises** In each chapter there are challenging exercises which may be solved by using one of the principles described in the Prologue: *Principles of Problem Solving*. Each such exercise is identified by the icon  and is accompanied by a hint that recommends a particular problem-solving principle to try. These exercises are intended to give students the opportunity to experience the process of problem-solving. (See for instance Exercises 2.7.94, 3.2.93, and 5.2.95.)
- **Now Try Exercise . . .** At the end of each example in the text the student is directed to one or more similar exercises in the section that help to reinforce the concepts and skills developed in that example.
- **Check Your Answer** Students are encouraged to check whether an answer they obtained is reasonable. This is emphasized throughout the text in numerous *Check Your Answer* sidebars that accompany the examples. (See for instance Examples 1.5.1 and 10, 1.7.1, and 9.1.7.)


A Complete Review Chapter We have included an extensive review chapter primarily as a handy reference for the basic concepts that are preliminary to this course.

- **Chapter 1 Fundamentals** This is the review chapter; it contains the fundamental concepts from algebra and analytic geometry that a student needs in order to begin a precalculus course. As much or as little of this chapter can be covered in class as needed, depending on the background of the students.
- **Diagnostic Test** The test at the end of Chapter 1 is designed as a diagnostic test for determining what parts of this review chapter need to be taught. It also serves to help students gauge exactly what topics they need to review.

Flexible Approach to Trigonometry The trigonometry chapters of this text have been written so that either the right triangle approach or the unit circle approach may be taught first. Putting these two approaches in different chapters, each with its relevant applications, helps to clarify the purpose of each approach. The chapters introducing trigonometry are as follows.

- **Chapter 5 Trigonometric Functions: Unit Circle Approach** This chapter introduces trigonometry through the unit circle. This approach emphasizes that the trigonometric functions are functions of real numbers, just like the polynomial and exponential functions with which students are already familiar.
- **Chapter 6 Trigonometric Functions: Right Triangle Approach** This chapter introduces trigonometry through right triangles. This approach builds on the foundation of a conventional high-school course in trigonometry.

Another way to teach trigonometry is to intertwine the two approaches. Some instructors teach this material in the following order: Sections 5.1, 5.2, 6.1, 6.2, 6.3, 5.3, 5.4, 5.5, 5.6, 6.4, 6.5, and 6.6. Our organization makes it easy to do this without obscuring the fact that the two approaches involve distinct representations of the same functions.

Graphing Devices We make use of graphing devices (graphing calculators, computers, and math apps) throughout the book. The examples in which technology is used are always preceded by examples in which students must graph or calculate by hand so that they can understand precisely what the device is doing when they later use it to simplify the routine part of their work. The exercises that require technology are marked with the special symbol ; these are optional and may be omitted without loss of continuity. (See *Technology in the Eighth Edition* following this preface, p. xix.)

Focus on Modeling The theme of modeling has been used throughout to unify and explain the many applications of precalculus. We have made a special effort to clarify the essential process of translating problems from words into the language of mathematics. (See, for instance, Examples 1.7.1 and 9.1.7.) Each chapter concludes with a *Focus on Modeling* section that highlights how the topics learned in the chapter are used in modeling real-world situations.

Review Sections and Chapter Tests Each chapter ends with an extensive review section that includes the following.

- **Properties and Formulas** The *Properties and Formulas* section at the end of each chapter contains a summary of the main formulas and procedures of the chapter.
- **Concept Check and Concept Check Answers** The *Concept Check* at the end of each chapter is designed to get the students to think about and explain each concept presented in the chapter and then to use the concept in a given problem. This provides a step-by-step review of all the main concepts in the chapter. Answers to the *Concept Check* questions are available at the book companion website.
- **Review Exercises** The *Review Exercises* at the end of each chapter recapitulate the basic concepts and skills of the chapter and include exercises that combine the different ideas learned in the chapter. Where appropriate, the last exercise is about matching equations with their graphs. The exercise requires students to relate properties of an equation to the corresponding properties of the graph.
- **Chapter Test** Each review section concludes with a *Chapter Test* designed to help students gauge their progress.
- **Cumulative Review Tests** *Cumulative Review Tests* following selected chapters are available at the book companion website. These tests contain problems that combine skills and concepts from the preceding chapters. The problems are intended to highlight the connections between the topics in these related chapters.
- **Answers** Brief answers to odd-numbered exercises in each section (including the *Review Exercises*) and to all questions in the *Concepts* exercises and *Chapter Tests*, are given in the back of the book.

Mathematical Vignettes Throughout the book we make use of the margins to provide historical notes, key insights, or applications of mathematics in the modern world. These serve to enliven the material and show that mathematics is an important, vital activity and that it is fundamental to everyday life.

- **Biographical Vignettes** These vignettes include biographies of interesting mathematicians and often include a key insight that the mathematician discovered. [See, for instance, the vignettes on Viète (Section 1.5), Turing (Section 2.6), and Thales (Section 6.6).]
- **Mathematics in the Modern World** This series of vignettes emphasizes the central role of mathematics in current advances in technology, the sciences, and the

social sciences. [See, for instance Unbreakable Codes (Section 3.6), Global Positioning System (Section 9.8), Looking Inside Your Head (Section 10.1), and Fair Division of Assets (Section 11.2).]

Book Companion Website A website that accompanies this book can be found at www.stewartmath.com. The site includes many resources for teaching precalculus, including the following.

- **Discovery Projects** *Discovery Projects* for each chapter are available at the book companion website. The projects are referenced in the text in the appropriate sections. Each project provides a challenging yet accessible set of activities that enable students (perhaps working in groups) to explore in greater depth an interesting aspect of the topic they have just learned (see, for instance, the Discovery Projects *Visualizing a Formula*, *Every Graph Tells a Story*, *Toricelli's Law*, *Mapping the World*, and *Will the Species Survive?*, referenced in Sections 1.3, 2.3, 3.1, 8.1, and 9.4).
- **Focus on Problem Solving** Several *Focus on Problem Solving* sections are available on the website. Each such section highlights one of the problem-solving principles introduced in the Prologue and includes several challenging problems. (See, for instance, *Recognizing Patterns*, *Using Analogy*, *Introducing Something Extra*, *Taking Cases*, and *Working Backward*.)
- **Cumulative Review Tests** *Cumulative Review Tests* following Chapters 4, 7, 10, and 12 are available at the book companion website.
- **Appendix B: Calculations and Significant Figures** This appendix, available at the book companion website, contains guidelines for rounding when working with approximate values.
- **Appendix C: Graphing with a Graphing Calculator** This appendix, available at the book companion website, includes general guidelines on graphing with a graphing calculator as well as guidelines on how to avoid common graphing pitfalls.
- **Appendix D: Using the TI-83/84 Graphing Calculator** In this appendix, available at the book companion website, we provide simple, easy-to-follow, step-by-step instructions for using the TI-83/84 graphing calculators.
- **Additional Topics** Several additional topics are available at the book companion website, including material on three-dimensional coordinate geometry, mathematics of finance, and probability and statistics.

Ancillaries

■ Instructor Resources

Additional instructor resources for this product are available online. Instructor assets include an Instructor's Manual, Educator's Guide, PowerPoint® slides, and a test bank powered by Cognero®. Sign up or sign in at <https://faculty.cengage.com/> to search for and access this product and its online resources.

Solutions and Answers Guide

The Solutions and Answers Guide provides worked-out solutions to all of the problems in the text. Located at the book companion website.

Instructor's Manual

The Instructor's Manual contains points to stress, suggested time to allot, text discussion topics, core materials for lecture, workshop/discussion suggestions, group work

exercises in a form suitable for handout, and suggested homework problems. Located at the book companion website.

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■ Student Resources

Student Solutions Manual

The Student Solutions Manual contains fully worked-out solutions to all odd-numbered exercises in the text, giving students a way to check their answers and ensure that they took the correct steps to arrive at an answer. Located at the book companion website.

Note-Taking Guide

The Note-Taking Guide is an innovative study aid that helps students develop a section-by-section summary of key concepts. Located at the book companion website.

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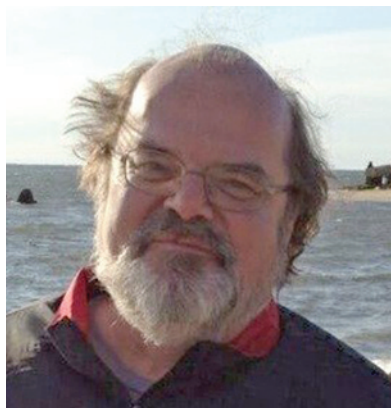
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A Tribute to Lothar Redlin



Lothar Redlin had an extraordinary talent for teaching mathematics. The classes that he taught were always full to capacity with students eager to learn from him. Lothar's presence in the classroom put everyone at ease, thus generating a wonderful learning atmosphere. Lothar held everyone's attention with his unassuming manner and his crystal clear explanations, which were drawn from his deep understanding of the meaning and purpose of the mathematics he was teaching.

Lothar attributed his vibrant mathematical intuition to the way his professors taught him; consequently, he always strived to transmit his personal insights into mathematics to his own students—in his teaching and in his textbooks—and his insights about pre-calculus continue to enhance this book. Lothar would complete teaching all the topics in a course well before the end of the school term, and his students would understand the subject exceptionally well—a tremendous witness to his unique way of understanding and explaining mathematics. Over the years, several of Lothar's former students, by then scientists, engineers, or financial analysts themselves, would contact him to discuss a mathematical problem that they came across in their work. Some of these discussions were the inspiration for applied exercises or discovery projects in this book.

Lothar grew up on Vancouver Island where he spent the summers helping his father, a professional fisherman, on their fishing boat (a 52-foot troller which Lothar had helped his father build). In the many hours they spent out at sea, Lothar's mind was not idle; he later recalled that the things he thought about were essentially mathematical (for instance, he figured out the mathematical basis for the Long-Range Navigation system they were using on the boat). Later, in high school, Lothar's talent for mathematics was recognized by his teachers and they encouraged him to attend the University of Victoria. He became the first member of his extended family to go to college.

I met Lothar when we were both graduate students in mathematics at McMaster University, and where James Stewart was one of our professors. Subsequently, we all worked together on various projects—first on research and later on writing textbooks. It has been my great fortune to have known and worked with Lothar for all the intervening years—until his untimely passing in 2018. Lothar's unique insights into mathematics, his humble and unassuming manner, his boundless generosity, and his earnest concern for everyone he met have enriched my life and the lives of his many students in inimitable ways; he is profoundly missed.

Saleem Watson


To the Student

This textbook was written for you to use as a guide to mastering precalculus mathematics. Here are some suggestions to help you get the most out of your course.

First of all, you should read the appropriate section of text *before* you attempt your homework problems. Reading a mathematics text is quite different from reading a novel, a newspaper, or even another textbook. You may find that you have to reread a passage several times before you understand it. Pay special attention to the examples, and work them out yourself with pencil and paper as you read. Then do the linked exercises referred to in “*Now Try Exercise . . .*” at the end of each example. With this kind of preparation you will be able to do your homework much more quickly and with more understanding.

Don’t make the mistake of trying to memorize every single rule or fact you may come across. Mathematics doesn’t consist simply of memorization. Mathematics is a *problem-solving art*, not just a collection of facts. To master the subject you must solve problems—lots of problems. Do as many of the exercises as you can. Be sure to write your solutions in a logical, step-by-step fashion. Don’t give up on a problem if you can’t solve it right away. Try to understand the problem more clearly—reread it thoughtfully and relate it to what you have learned from your teacher and from the examples in the text. Struggle with it until you solve it. Once you have done this a few times you will begin to understand what mathematics is really all about.

Answers to the odd-numbered exercises, as well as all the answers (even and odd) to the concept exercises and chapter tests, appear at the back of the book. If your answer differs from the one given, don’t immediately assume that you are wrong. There may be a calculation that connects the two answers and makes both correct. For example, if you get $1/(\sqrt{2} - 1)$ but the answer given is $1 + \sqrt{2}$, your answer *is* correct, because you can multiply both numerator and denominator of your answer by $\sqrt{2} + 1$ to change it to the given answer. In rounding approximate answers, follow the guidelines in Appendix B: *Calculations and Significant Figures*.

The symbol  is used to warn against committing an error. We have placed this symbol in the margin to point out situations where we have found that many of our students make the same mistake.

Abbreviations

The following abbreviations are used in the text.


| | | | | | |
|-------------|-------------|------------|------------|------------|----------------------------|
| cm | centimeter | kPa | kilopascal | N | Newton |
| dB | decibel | L | liter | psi | pounds per in ² |
| dL | deci-liter | lb | pound | qt | quart |
| F | farad | lm | lumen | oz | ounce |
| ft | foot | mol | mole | s | second |
| g | gram | m | meter | Ω | ohm |
| gal | gallon | mg | milligram | V | volt |
| h | hour | MHz | megahertz | W | watt |
| H | henry | mi | mile | yd | yard |
| Hz | Hertz | min | minute | yr | year |
| in. | inch | mL | milliliter | °C | degree Celsius |
| J | Joule | mm | millimeter | °F | degree Fahrenheit |
| kcal | kilocalorie | MW | megawatts | K | Kelvin |
| kg | kilogram | nm | nanometers | ⇒ | implies |
| km | kilometer | Å | angstrom | ⇔ | is equivalent to |

Technology in the Eighth Edition

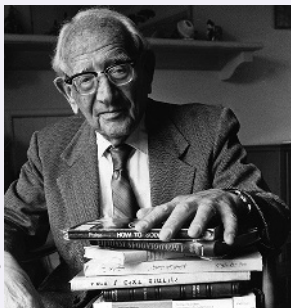
Several different technology resources are available for computing and graphing. These include graphing and computing tools on the Internet, math apps for smartphones and tablets, and graphing calculators. In many situations, using technology eliminates the routine work of computing and graphing, thus allowing us to more sharply focus our attention on understanding the concepts of precalculus. For instance, we can use technology to quickly graph a function, modify the function and graph it again, in order to gain insight into the behavior of the function.

There are graphing and computing programs available free of charge on the Internet or as apps for smartphones or tablets. These include Desmos, GeoGebra, WolframAlpha, and others. Of course, graphing calculators can also be used to perform all the needed computing and graphing in the exercises, although many of the web-based math apps are more intuitive to use.



In this edition, rather than refer to a specific math app or graphing device, we use the icon  to indicate that technology is needed to complete the exercise. For such exercises you are expected to use technology to draw a graph or perform a calculation. You are encouraged to use technology on other exercises as well, to check your work or to explore related problems.

Principles of Problem Solving



AP Images

GEORGE POLYA (1887–1985) is famous among mathematicians for his ideas on problem-solving. His lectures on problem-solving at Stanford University attracted overflow crowds, and he held his audience on the edges of their seats, leading them to discover solutions for themselves. He was able to do this because of his deep insight into the psychology of problem-solving. His well-known book *How To Solve It* has been translated into 15 languages. He said that Euler (see Section 1.6) was unique among great mathematicians because he explained how he found his results. Polya often said to his students and colleagues, “Yes, I see that your proof is correct, but how did you discover it?” In the preface to *How To Solve It*, Polya writes, “A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.”

The ability to solve problems is a highly prized skill in many aspects of our lives; it is certainly an important part of any mathematics course. There are no hard and fast rules that will ensure success in solving problems. However, in this prologue we outline some general steps in the problem-solving process and we give principles that are useful in solving certain types of problems. These steps and principles are just common sense made explicit. They have been adapted from George Polya’s insightful book *How To Solve It*.

1. Understand the Problem

The first step is to read the problem and make sure that you understand it. Ask yourself the following questions:

- What is the unknown?
- What are the given quantities?
- What are the given conditions?

For many problems it is useful to

draw a diagram

and identify the given and required quantities on the diagram. Usually, it is necessary to

introduce suitable notation

In choosing symbols for the unknown quantities, we often use letters such as a , b , c , m , n , x , and y , but in some cases it helps to use initials as suggestive symbols, for instance, V for volume or t for time.

2. Think of a Plan

Find a connection between the given information and the unknown that enables you to calculate the unknown. It often helps to ask yourself explicitly: “How can I relate the given to the unknown?” If you don’t see a connection immediately, the following ideas may be helpful in devising a plan.

■ Try to Recognize Something Familiar

Relate the given situation to previous knowledge. Look at the unknown and try to recall a more familiar problem that has a similar unknown.

■ Try to Recognize Patterns

Certain problems are solved by recognizing that some kind of pattern is occurring. The pattern could be geometric, numerical, or algebraic. If you can see regularity or repetition in a problem, then you might be able to guess what the pattern is and then prove it.

■ Use Analogy

Try to think of an analogous problem, that is, a similar or related problem but one that is easier than the original. If you can solve the similar, simpler problem, then it might give you the clues you need to solve the original, more difficult one. For instance, if a problem involves very large numbers, you could first try a similar problem with smaller numbers. Or if the problem is in three-dimensional geometry, you could look

for something similar in two-dimensional geometry. Or if the problem you start with is a general one, you could first try a special case.

■ Introduce Something Extra

You may sometimes need to introduce something new—an auxiliary aid—to make the connection between the given and the unknown. For instance, in a problem for which a diagram is useful, the auxiliary aid could be a new line drawn in the diagram. In a more algebraic problem the aid could be a new unknown that relates to the original unknown.

■ Take Cases

You may sometimes have to split a problem into several cases and give a different argument for each case. We often have to use this strategy in dealing with absolute value.

■ Work Backward

Sometimes it is useful to imagine that your problem is solved and work backward, step by step, until you arrive at the given data. Then you might be able to reverse your steps and thereby construct a solution to the original problem. This procedure is commonly used in solving equations. For instance, in solving the equation $3x - 5 = 7$, we suppose that x is a number that satisfies $3x - 5 = 7$ and work backward. We add 5 to each side of the equation and then divide each side by 3 to get $x = 4$. Since each of these steps can be reversed, we have solved the problem.

■ Establish Subgoals

In a complex problem it is often useful to set subgoals (in which the desired situation is only partially fulfilled). If you can attain or accomplish these subgoals, then you may be able to build on them to reach your final goal.

■ Indirect Reasoning

Sometimes it is appropriate to attack a problem indirectly. In using **proof by contradiction** to prove that P implies Q , we assume that P is true and Q is false and try to see why this cannot happen. Somehow we have to use this information and arrive at a contradiction to what we absolutely know is true.

■ Mathematical Induction

In proving statements that involve a positive integer n , it is frequently helpful to use the Principle of Mathematical Induction, which is discussed in Section 11.4.

3. Carry Out the Plan

In Step 2, a plan was devised. In carrying out that plan, you must check each stage of the plan and write the details that prove that each stage is correct.

4. Look Back

Having completed your solution, it is wise to look back over it, partly to see whether any errors have been made and partly to see whether you can discover an easier way to solve the problem. Looking back also familiarizes you with the method of solution, which may be useful for solving a future problem. Descartes said, “Every problem that I solved became a rule which served afterwards to solve other problems.”

We illustrate some of these principles of problem-solving with an example.

Problem ■ Average Speed

A driver sets out on a journey. For the first half of the distance, the driver's speed is 30 mi/h; during the second half the speed is 60 mi/h. What is the average speed on this trip?

Thinking About the Problem

It is tempting to take the average of the speeds and say that the average speed for the entire trip is $\frac{30 + 60}{2} = 45$ mi/h. But is this approach really correct?

Try a special case. ►

Let's look at an easily calculated special case. Suppose that the total distance traveled is 120 mi. Since the first 60 mi is traveled at 30 mi/h, it takes 2 h. The second 60 mi is traveled at 60 mi/h, so it takes one hour. Thus, the total time is $2 + 1 = 3$ hours and the average speed is $\frac{120}{3} = 40$ mi/h. So our guess of 45 mi/h was wrong.

Solution

Understand the problem. ►

We need to look more carefully at the meaning of average speed. It is defined as

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

Introduce notation. ►

Let d be the distance traveled on each half of the trip. Let t_1 and t_2 be the times taken for the first and second halves of the trip. Now we can write down the information we have been given. For the first and second halves of the trip we have

State what is given. ►

$$30 = \frac{d}{t_1} \quad 60 = \frac{d}{t_2}$$

Identify the unknown. ►

Now we identify the quantity that we are asked to find:

$$\text{average speed for entire trip} = \frac{\text{total distance}}{\text{total time}} = \frac{2d}{t_1 + t_2}$$

Connect the given with the unknown. ►

To calculate this quantity, we need to know t_1 and t_2 , so we solve the above equations for these times:

$$t_1 = \frac{d}{30} \quad t_2 = \frac{d}{60}$$

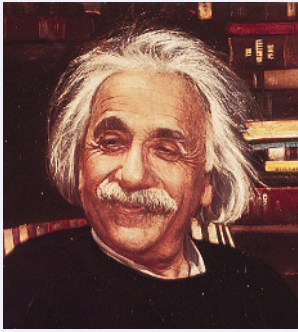
Now we have the ingredients needed to calculate the desired quantity:

$$\begin{aligned} \text{average speed} &= \frac{2d}{t_1 + t_2} = \frac{2d}{\frac{d}{30} + \frac{d}{60}} \\ &= \frac{60(2d)}{60\left(\frac{d}{30} + \frac{d}{60}\right)} && \text{Multiply numerator and denominator by 60} \\ &= \frac{120d}{2d + d} = \frac{120d}{3d} = 40 \end{aligned}$$

So the average speed for the entire trip is 40 mi/h.

Look back ►

This answer agrees with the answer we obtained for the special cases we tried above in *Thinking About the Problem*. ■



Bettmann/Corbis

Don't feel bad if you can't solve these problems right away. Problems 1 and 4 were sent to Albert Einstein by his friend Wertheimer. Einstein (and his friend Bucky) enjoyed the problems and wrote back to Wertheimer. Here is part of his reply:

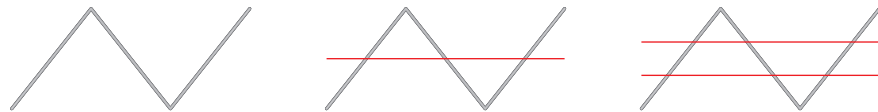
Your letter gave us a lot of amusement. The first intelligence test fooled both of us (Bucky and me). Only on working it out did I notice that no time is available for the downhill run! Mr. Bucky was also taken in by the second example, but I was not. Such drolleries show us how stupid we are!

(See *Mathematical Intelligencer*, Spring 1990, page 41.)



■ Problems

1. **Distance, Time, and Speed** An old car has to travel a 2-mile route, uphill and down. Because it is so old, the car can climb the first mile—the ascent—no faster than an average speed of 15 mi/h. How fast does the car have to travel the second mile—on the descent it can go faster, of course—to achieve an average speed of 30 mi/h for the trip?
2. **Comparing Discounts** Which price is better for the buyer, a 40% discount or two successive discounts of 20%?
3. **Cutting up a Wire** A piece of wire is bent as shown in the figure. You can see that one cut through the wire (represented by the red lines) produces four pieces and two parallel cuts produce seven pieces. How many pieces will be produced by 142 parallel cuts? Write a formula for the number of pieces produced by n parallel cuts.



4. **Amoeba Propagation** An amoeba propagates by simple division; each split takes 3 minutes to complete. When such an amoeba is put into a glass container with a nutrient fluid, the container is full of amoebas in one hour. How long would it take for the container to be filled if we start with not one amoeba, but two?
5. **Batting Averages** Player A has a higher batting average than player B for the first half of the baseball season. Player A also has a higher batting average than player B for the second half of the season. Is it necessarily true that player A has a higher batting average than player B for the entire season?
6. **Coffee and Cream** A spoonful of cream is taken from a pitcher of cream and put into a cup of coffee. The coffee is stirred. Then a spoonful of this mixture is put into the pitcher of cream. Is there now more cream in the coffee cup or more coffee in the pitcher of cream?
7. **Wrapping the World** A ribbon is tied tightly around the earth at the equator. How much more ribbon would you need if you raised the ribbon 1 ft above the equator everywhere? (You don't need to know the radius of the earth to solve this problem.)
8. **Ending Up Where You Started** A woman starts at a point P on the earth's surface and walks 1 mi south, then 1 mi east, then 1 mi north, and finds herself back at P , the starting point. Describe all points P for which this is possible. [*Hint:* There are infinitely many such points, all but one of which lie in Antarctica.]



You can apply these problem-solving principles in solving any problem or exercise. The icon next to an exercise indicates that one or more of the strategies discussed here is particularly useful for solving the exercise. You can find more examples and problems that highlight these principles at the book companion website www.stewartmath.com.



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1

Fundamentals

- 1.1 Real Numbers
- 1.2 Exponents and Radicals
- 1.3 Algebraic Expressions
- 1.4 Rational Expressions
- 1.5 Equations
- 1.6 Complex Numbers
- 1.7 Modeling with Equations
- 1.8 Inequalities
- 1.9 The Coordinate Plane;
Graphs of Equations;
Circles
- 1.10 Lines
- 1.11 Solving Equations and
Inequalities Graphically
- 1.12 Modeling Variation
 - Focus on Modeling
 - Fitting Lines to Data

In this first chapter we review the real numbers, equations, and the coordinate plane. You are probably already familiar with these concepts, but it is useful to get a fresh look at how these ideas work together to help you solve problems and model (or describe) real-world situations.

In the *Focus on Modeling* at the end of the chapter we learn how to find linear trends in data and how to use these trends to make predictions about the object or process being modeled.

1.1 Real Numbers

- Real Numbers
- Properties of Real Numbers
- Addition and Subtraction
- Multiplication and Division
- The Real Line
- Sets and Intervals
- Absolute Value and Distance

In the real world we use numbers to measure and compare quantities. For example, we measure temperature, length, height, weight, distance, speed, acceleration, energy, force, angles, pressure, cost, and so on. Figure 1 illustrates some situations in which numbers are used. Numbers also allow us to express relationships between different quantities—for example, relationships between the radius and volume of a ball, between miles driven and gas used, or between education level and starting salary.



Length

Weight

Speed

Pressure

Figure 1 | Measuring with real numbers

Real Numbers

Let’s review the types of numbers that make up the real number system. We start with the **natural numbers**:

$$1, 2, 3, 4, \dots$$

The **integers** consist of the natural numbers together with their negatives and 0:

$$\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots$$

We construct the **rational numbers** by taking ratios of integers. Thus any rational number r can be expressed as

$$r = \frac{m}{n}$$

where m and n are integers and $n \neq 0$. Examples are

$$\frac{1}{2}, \quad -\frac{3}{7}, \quad 46 = \frac{46}{1}, \quad 0.17 = \frac{17}{100}$$

(Recall that division by 0 is always ruled out, so expressions like $\frac{3}{0}$ and $\frac{0}{0}$ are undefined.) There are also real numbers, such as $\sqrt{2}$, that cannot be expressed as a ratio of integers and are therefore called **irrational numbers**. It can be shown, with varying degrees of difficulty, that these numbers are also irrational:

$$\sqrt{3}, \quad \sqrt{5}, \quad \sqrt[3]{2}, \quad \pi, \quad \frac{3}{\pi^2}$$

The set of all real numbers is usually denoted by the symbol \mathbb{R} . When we use the word *number* without qualification, we will mean “real number.” Figure 2 is a diagram of the types of real numbers that we work with in this book.

Every real number has a decimal representation. If the number is rational, then its corresponding decimal is repeating. For example,

$$\frac{1}{2} = 0.5000\dots = 0.5\bar{0} \qquad \frac{2}{3} = 0.66666\dots = 0.\bar{6}$$

$$\frac{157}{495} = 0.3171717\dots = 0.31\bar{7} \qquad \frac{9}{7} = 1.285714285714\dots = 1.28571\bar{4}$$

The different types of real numbers were invented to meet specific needs. For example, natural numbers are needed for counting, negative numbers for describing debt or below-zero temperatures, rational numbers for concepts like “half a gallon of milk,” and irrational numbers for measuring certain distances, like the diagonal of a square.

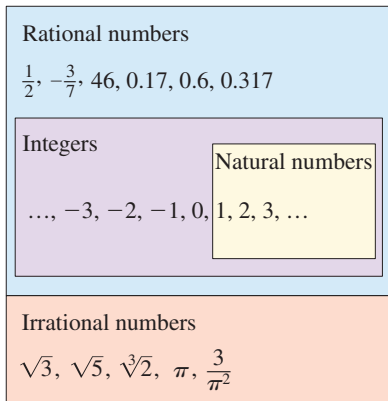


Figure 2 | The real number system

A repeating decimal such as

$$x = 3.5474747\ldots$$

is a rational number. To convert it to a ratio of two integers, we write

$$\begin{array}{r} 1000x = 3547.47474747\ldots \\ 10x = 35.47474747\ldots \\ \hline 990x = 3512.0 \end{array}$$

Thus $x = \frac{3512}{990}$. (The idea is to multiply x by appropriate powers of 10 and then subtract to eliminate the repeating part.) See also Example 11.3.7 and Exercise 11.3.77.

(The bar indicates that the sequence of digits repeats indefinitely.) If a given number is irrational, its decimal representation is nonrepeating:

$$\sqrt{2} = 1.414213562373095\ldots \quad \pi = 3.141592653589793\ldots$$

If we stop the decimal expansion of any number at a certain place, we get an approximation to the number. For instance, we can write

$$\pi \approx 3.14159265$$

where the symbol \approx is read “is approximately equal to.” The more decimal places we retain, the better our approximation.

■ Properties of Real Numbers

We all know that $2 + 3 = 3 + 2$, and $5 + 7 = 7 + 5$, and $513 + 87 = 87 + 513$, and so on. In algebra we express all these (infinitely many) facts by writing

$$a + b = b + a$$

where a and b stand for any two numbers. In other words, “ $a + b = b + a$ ” is a concise way of saying that “when we add two numbers, the order of addition doesn’t matter.” This fact is called the *Commutative Property* of addition. From our experience with numbers we know that the properties in the following box are also valid.

| Properties of Real Numbers | | |
|-------------------------------|---|--|
| Property | Example | Description |
| Commutative Properties | | |
| $a + b = b + a$ | $7 + 3 = 3 + 7$ | When we add two numbers, order doesn’t matter. |
| $ab = ba$ | $3 \cdot 5 = 5 \cdot 3$ | When we multiply two numbers, order doesn’t matter. |
| Associative Properties | | |
| $(a + b) + c = a + (b + c)$ | $(2 + 4) + 7 = 2 + (4 + 7)$ | When we add three numbers, it doesn’t matter which two we add first. |
| $(ab)c = a(bc)$ | $(3 \cdot 7) \cdot 5 = 3 \cdot (7 \cdot 5)$ | When we multiply three numbers, it doesn’t matter which two we multiply first. |
| Distributive Property | | |
| $a(b + c) = ab + ac$ | $2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5$ | When we multiply a number by a sum of two numbers, we get the same result as we get if we multiply the number by each of the terms and then add the results. |
| $(b + c)a = ab + ac$ | $(3 + 5) \cdot 2 = 2 \cdot 3 + 2 \cdot 5$ | |

The Distributive Property applies whenever we multiply a number by a sum. Figure 3 explains why this property works for the case in which all the numbers are positive integers, but the property is true for any real numbers a , b , and c .

The Distributive Property is crucial because it describes the way addition and multiplication interact with each other.

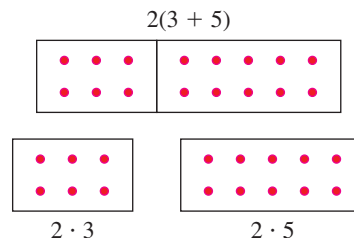


Figure 3 | The Distributive Property

Example 1 ■ Using the Distributive Property

$$\begin{aligned} \text{(a)} \quad 2(x + 3) &= 2 \cdot x + 2 \cdot 3 \\ &= 2x + 6 \end{aligned}$$

Distributive Property

Simplify

$$\begin{aligned} \text{(b)} \quad (a + b)(x + y) &= (a + b)x + (a + b)y \\ &= (ax + bx) + (ay + by) \\ &= ax + bx + ay + by \end{aligned}$$

Distributive Property

Distributive Property

Associative Property of Addition

In the last step we removed all the parentheses because, according to the Associative Property, the order of addition doesn't matter.



Now Try Exercise 15

Don't assume that $-a$ is a negative number. Whether $-a$ is negative or positive depends on the value of a . For example, if $a = 5$, then $-a = -5$, a negative number, but if $a = -5$, then $-a = -(-5) = 5$ (Property 2), a positive number.

■ Addition and Subtraction

The number 0 is special for addition; it is called the **additive identity** because $a + 0 = a$ for any real number a . Every real number a has a **negative**, $-a$, that satisfies $a + (-a) = 0$. **Subtraction** is the operation that undoes addition; to subtract a number from another, we simply add the negative of that number. By definition

$$a - b = a + (-b)$$

To combine real numbers involving negatives, we use the following properties.

Properties of Negatives**Property**

1. $(-1)a = -a$

2. $-(-a) = a$

3. $(-a)b = a(-b) = -(ab)$

4. $(-a)(-b) = ab$

5. $-(a + b) = -a - b$

6. $-(a - b) = b - a$

Example

$(-1)5 = -5$

$-(-5) = 5$

$(-5)7 = 5(-7) = -(5 \cdot 7)$

$(-4)(-3) = 4 \cdot 3$

$-(3 + 5) = -3 - 5$

$-(5 - 8) = 8 - 5$

Properties 5 and 6 follow from the Distributive Property.

Property 6 states the intuitive fact that $a - b$ and $b - a$ are negatives of each other. Property 5 is often used with more than two terms:

$$-(a + b + c) = -a - b - c$$

Example 2 ■ Using Properties of Negatives

Let x , y , and z be real numbers.

$$\text{(a)} \quad -(x + 2) = -x - 2$$

Property 5: $-(a + b) = -a - b$

$$\text{(b)} \quad -(x + y - z) = -x - y - (-z)$$

Property 5: $-(a + b) = -a - b$

$$= -x - y + z$$

Property 2: $-(-a) = a$ 

Now Try Exercise 23

■ Multiplication and Division

The number 1 is special for multiplication; it is called the **multiplicative identity** because $a \cdot 1 = a$ for any real number a . Every nonzero real number a has an **inverse**, $1/a$, that satisfies $a \cdot (1/a) = 1$. **Division** is the operation that undoes multiplication; to divide by a number, we multiply by the inverse of that number. If $b \neq 0$, then, by definition,

$$a \div b = a \cdot \frac{1}{b}$$

We write $a \cdot (1/b)$ as simply a/b . We refer to a/b as the **quotient** of a and b or as the **fraction** a over b ; a is the **numerator** and b is the **denominator** (or **divisor**). To combine real numbers using the operation of division, we use the following properties.

Properties of Fractions

| Property | Example | Description |
|---|--|---|
| 1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ | $\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$ | When multiplying fractions , multiply numerators and denominators. |
| 2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ | $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$ | When dividing fractions , invert the divisor and multiply the numerators and denominators. |
| 3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ | $\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$ | When adding fractions with the same denominator , add the numerators. |
| 4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ | $\frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7 + 3 \cdot 5}{35} = \frac{29}{35}$ | When adding fractions with different denominators , find a common denominator. Then add the numerators. |
| 5. $\frac{ac}{bc} = \frac{a}{b}$ | $\frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$ | Cancel numbers that are common factors in numerator and denominator. |
| 6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$ | $\frac{2}{3} = \frac{6}{9}$, so $2 \cdot 9 = 3 \cdot 6$ | Cross-multiply . |

When adding fractions with different denominators, we don't usually use Property 4. Instead we rewrite the fractions so that they have the smallest possible common denominator (often smaller than the product of the denominators), and then we use Property 3. This denominator is the **Least Common Denominator (LCD)** described in the next example.

Example 3 ■ Using the LCD to Add Fractions

Evaluate: $\frac{5}{36} + \frac{7}{120}$

Solution Factoring each denominator into prime factors gives

$$36 = 2^2 \cdot 3^2 \quad \text{and} \quad 120 = 2^3 \cdot 3 \cdot 5$$

We find the least common denominator (LCD) by forming the product of all the prime factors that occur in these factorizations, using the highest power of each prime factor. Thus the LCD is $2^3 \cdot 3^2 \cdot 5 = 360$. So

$$\frac{5}{36} + \frac{7}{120} = \frac{5 \cdot 10}{36 \cdot 10} + \frac{7 \cdot 3}{120 \cdot 3} \quad \text{Use common denominator}$$

$$= \frac{50}{360} + \frac{21}{360} = \frac{71}{360} \quad \text{Property 3: Adding fractions with the same denominator}$$



Now Try Exercise 29

■ The Real Line

The real numbers can be represented by points on a line, as shown in Figure 4. The positive direction (toward the right) is indicated by an arrow. We choose an arbitrary reference point O , called the **origin**, which corresponds to the real number 0. Given any convenient unit of measurement, each positive number x is represented by the point on the line a distance of x units to the right of the origin, and the corresponding negative number $-x$ is represented by the point x units to the left of the origin. The number associated with the point P is called the **coordinate** of P , and the line is then called a **coordinate line**, or a **real number line**, or simply a **real line**. Often we identify the point with its coordinate and think of a number as being a point on the real line.

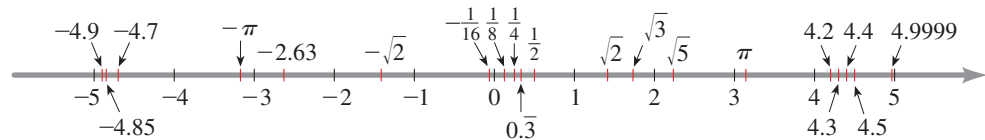


Figure 4 | The real line

The real numbers are *ordered*. We say that **a is less than b** and write $a < b$ if $b - a$ is a positive number. Geometrically, this means that a lies to the left of b on the number line. Equivalently, we can say that **b is greater than a** and write $b > a$. The symbol $a \leq b$ (or $b \geq a$) means that either $a < b$ or $a = b$ and is read “ a is less than or equal to b .” For instance, the following are true inequalities (see Figure 4):

$$-5 < -4.9 \quad -\pi < -3 \quad \sqrt{2} < 2 \quad 4 < 4.4 < 4.9999$$

■ Sets and Intervals

A **set** is a collection of objects, and these objects are called the **elements** of the set. If S is a set, the notation $a \in S$ means that a is an element of S , and $b \notin S$ means that b is not an element of S . For example, if Z represents the set of integers, then $-3 \in Z$ but $\pi \notin Z$.

Some sets can be described by listing their elements within braces. For instance, the set A that consists of all positive integers less than 7 can be written as

$$A = \{1, 2, 3, 4, 5, 6\}$$

We could also write A in **set-builder notation** as

$$A = \{x \mid x \text{ is an integer and } 0 < x < 7\}$$

which is read “ A is the set of all x such that x is an integer and $0 < x < 7$.”

If S and T are sets, then their **union** $S \cup T$ is the set that consists of all elements that are in S or T (or in both). The **intersection** of S and T is the set $S \cap T$ consisting of all



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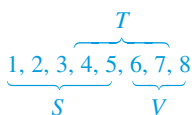
Discovery Project ■ Real Numbers in the Real World

Real-world measurements often involve units. For example, we usually measure distance in feet, miles, centimeters, or kilometers. Some measurements involve different types of units. For example, speed is measured in miles per hour or meters per second. We often need to convert a measurement from one type of unit to another. In this project we explore types of units used for different purposes and how to convert from one type of unit to another. You can find the project at www.stewartmath.com.

elements that are in both S and T . In other words, $S \cap T$ is the common part of S and T . The **empty set**, denoted by \emptyset , is the set that contains no element.

Example 4 ■ Union and Intersection of Sets

If $S = \{1, 2, 3, 4, 5\}$, $T = \{4, 5, 6, 7\}$, and $V = \{6, 7, 8\}$, find the sets $S \cup T$, $S \cap T$, and $S \cap V$.



Solution

- $S \cup T = \{1, 2, 3, 4, 5, 6, 7\}$ All elements in S or T
- $S \cap T = \{4, 5\}$ Elements common to both S and T
- $S \cap V = \emptyset$ S and V have no element in common



Now Try Exercise 41

Certain sets of real numbers, called **intervals**, occur frequently in calculus and correspond geometrically to line segments. If $a < b$, then the **open interval** from a to b consists of all numbers between a and b and is denoted (a, b) . The **closed interval** from a to b includes the endpoints and is denoted $[a, b]$. Using set-builder notation, we can write

$$(a, b) = \{x \mid a < x < b\} \quad [a, b] = \{x \mid a \leq x \leq b\}$$



Figure 5 | The open interval (a, b)



Figure 6 | The closed interval $[a, b]$

Note that parentheses $()$ in the interval notation and open circles on the graph in Figure 5 indicate that endpoints are *excluded* from the interval, whereas square brackets $[\]$ and solid circles in Figure 6 indicate that the endpoints are *included*. Intervals may also include one endpoint but not the other, or they may extend infinitely far in one direction or both directions. The following table lists the possible types of intervals.

The symbol ∞ (“infinity”) does not stand for a number. The notation (a, ∞) , for instance, simply indicates that the interval has no endpoint on the right but extends infinitely far in the positive direction.

| Notation | Set Description | Graph |
|---------------------|--|-------|
| (a, b) | $\{x \mid a < x < b\}$ | |
| $[a, b]$ | $\{x \mid a \leq x \leq b\}$ | |
| $[a, b)$ | $\{x \mid a \leq x < b\}$ | |
| $(a, b]$ | $\{x \mid a < x \leq b\}$ | |
| (a, ∞) | $\{x \mid a < x\}$ | |
| $[a, \infty)$ | $\{x \mid a \leq x\}$ | |
| $(-\infty, b)$ | $\{x \mid x < b\}$ | |
| $(-\infty, b]$ | $\{x \mid x \leq b\}$ | |
| $(-\infty, \infty)$ | \mathbb{R} (set of all real numbers) | |

Example 5 ■ Graphing Intervals

Express each interval in terms of inequalities, and then graph the interval.

- (a) $[-1, 2) = \{x \mid -1 \leq x < 2\}$
- (b) $[1.5, 4] = \{x \mid 1.5 \leq x \leq 4\}$
- (c) $(-3, \infty) = \{x \mid -3 < x\}$



Now Try Exercise 47

No Smallest or Largest Number in an Open Interval

Any interval contains infinitely many numbers—every point on the graph of an interval corresponds to a real number. In the closed interval $[0, 1]$, the smallest number is 0 and the largest is 1, but the open interval $(0, 1)$ contains no smallest or largest number. To see this, note that 0.01 is close to zero, but 0.001 is closer, 0.0001 is closer yet, and so on. We can always find a number in the interval $(0, 1)$ closer to zero than any given number. Since 0 itself is not in the interval, the interval contains no smallest number. Similarly, 0.99 is close to 1, but 0.999 is closer, 0.9999 closer yet, and so on. Since 1 itself is not in the interval, the interval has no largest number.

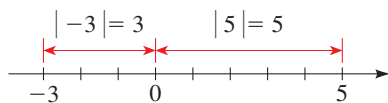
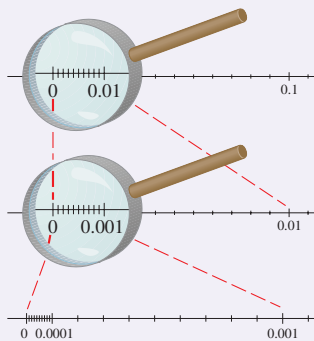


Figure 9

Example 6 ■ Finding Unions and Intersections of Intervals

Graph each set.

(a) $(1, 3) \cap [2, 7]$ (b) $(1, 3) \cup [2, 7]$

Solution

- (a) The intersection of two intervals consists of the numbers that are in both intervals; geometrically, this is where the intervals overlap. Therefore

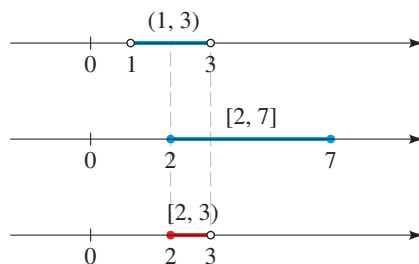
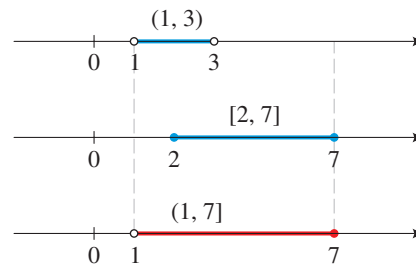
$$\begin{aligned}(1, 3) \cap [2, 7] &= \{x \mid 1 < x < 3 \text{ and } 2 \leq x \leq 7\} \\ &= \{x \mid 2 \leq x < 3\} = [2, 3)\end{aligned}$$

This set is illustrated in Figure 7.

- (b) The union of two intervals consists of the numbers that are in either one interval or the other (or both). Therefore

$$\begin{aligned}(1, 3) \cup [2, 7] &= \{x \mid 1 < x < 3 \text{ or } 2 \leq x \leq 7\} \\ &= \{x \mid 1 < x \leq 7\} = (1, 7]\end{aligned}$$

This set is illustrated in Figure 8.

Figure 7 | $(1, 3) \cap [2, 7] = [2, 3)$ Figure 8 | $(1, 3) \cup [2, 7] = (1, 7]$

Now Try Exercise 61

■ Absolute Value and Distance

The **absolute value** of a number a , denoted by $|a|$, is the distance from a to 0 on the real number line (see Figure 9). Distance is always positive or zero, so we have $|a| \geq 0$ for every number a . Remembering that $-a$ is positive when a is negative, we have the following definition.

Definition of Absolute ValueIf a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example 7 ■ Evaluating Absolute Values of Numbers

- (a) $|3| = 3$
 (b) $|-3| = -(-3) = 3$
 (c) $|0| = 0$
 (d) $|3 - \pi| = -(3 - \pi) = \pi - 3$ (since $3 < \pi \Rightarrow 3 - \pi < 0$)

Now Try Exercise 67

When working with absolute values, we use the following properties.

| Properties of Absolute Value | | |
|---|--|--|
| Property | Example | Description |
| 1. $ a \geq 0$ | $ -3 = 3 \geq 0$ | The absolute value of a number is always positive or zero. |
| 2. $ a = -a $ | $ 5 = -5 $ | A number and its negative have the same absolute value. |
| 3. $ ab = a b $ | $ -2 \cdot 5 = -2 5 $ | The absolute value of a product is the product of the absolute values. |
| 4. $\left \frac{a}{b} \right = \frac{ a }{ b }$ | $\left \frac{12}{-3} \right = \frac{ 12 }{ -3 }$ | The absolute value of a quotient is the quotient of the absolute values. |
| 5. $ a + b \leq a + b $ | $ -3 + 5 \leq -3 + 5 $ | Triangle Inequality |

What is the distance on the real line between the numbers -2 and 11 ? From Figure 10 we see that the distance is 13 . We arrive at this by finding either $|11 - (-2)| = 13$ or $|(-2) - 11| = 13$. From this observation we make the following definition.

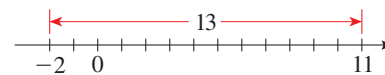
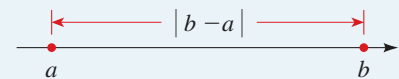


Figure 10

Distance between Points on the Real Line

If a and b are real numbers, then the **distance** between the points a and b on the real line is

$$d(a, b) = |b - a|$$



From Property 6 of negatives it follows that

$$|b - a| = |a - b|$$

This confirms that, as we would expect, the distance from a to b is the same as the distance from b to a .

Example 8 ■ Distance Between Points on the Real Line

The distance between the numbers 2 and -8 is

$$d(a, b) = |-8 - 2| = |-10| = 10$$

We can check this calculation geometrically, as shown in Figure 11.

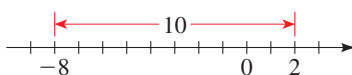


Figure 11