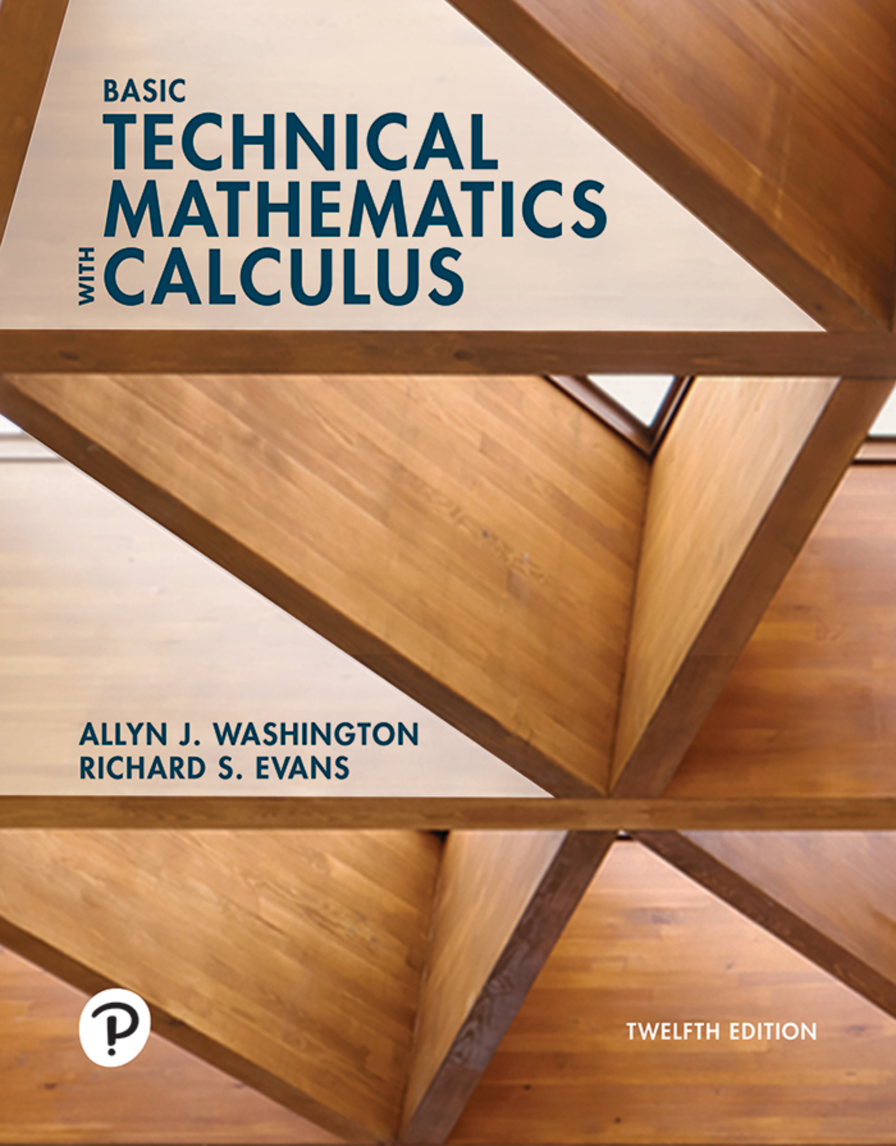

BASIC TECHNICAL MATHEMATICS WITH CALCULUS

ALLYN J. WASHINGTON
RICHARD S. EVANS

TWELFTH EDITION

TWELFTH EDITION

Basic Technical Mathematics with Calculus

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In Memory of Allyn Washington
1930 – 2020

Teacher, scholar, author, friend.
Warm and generous, family man.

Committed to his work, this book.
His love for life, never shook.

Baseball, bridge, wartime tales,
Brought him joy, filled his sails.

His open heart and smiling face,
Live on inside for all my days.

Rich Evans

Contents

Preface

Scope of the Book

Basic Technical Mathematics with Calculus, Twelfth Edition, is intended primarily for students in technical and pre-engineering technical programs or other programs for which coverage of mathematics is required. Chapters 1 through 20 provide the necessary background for further study with an integrated treatment of algebra and trigonometry. Chapter 21 covers the basic topics of analytic geometry, and Chapter 22 gives an introduction to statistics. Chapters 23 through 31 cover fundamental concepts of calculus including limits, derivatives, integrals, series representation of functions, and differential equations. In the examples and exercises, numerous applications from the various fields of technology are included, primarily to indicate where and how mathematical techniques are used. However, it is not necessary that the student have a specific knowledge of the technical area from which any given problem is taken. Most students using this text will have a background that includes some algebra and geometry. However, the material is presented in adequate detail for those who may need more study in these areas. The material presented here is sufficient for two to three semesters. One of the principal reasons for the arrangement of topics in this text is to present material in an order that allows a student to take courses concurrently in allied technical areas, such as physics and electricity. These allied courses normally require a student to know certain mathematics topics by certain definite times; yet the traditional order of topics in mathematics courses makes it difficult to attain this coverage without loss of continuity. However, the material in this book can be rearranged to fit any appropriate sequence of topics. The approach used in this text is not unduly rigorous mathematically, although all appropriate terms and concepts are introduced as needed and given an intuitive or algebraic foundation. The aim is to help the student develop an understanding of mathematical methods without simply providing a collection of formulas. The text material is developed recognizing that it is essential for the student to have a sound background in algebra and trigonometry in order to understand and succeed in any subsequent work in mathematics.

New to This Edition

The focus of this revision was to address feedback from our many MyLab Math users. Here are details of what has been updated in the text and online:

- Within the text, we addressed feedback received from users and reviewers. We also updated real-world data and scenarios to bring them up to date.
- We conducted an external review of the text's content to determine how it could be improved to address issues related to diversity, equity, and inclusion. The results of that review informed the revision.
- At the request of users, we added Appendix E, which covers Binary and Hexadecimal Numbers. This appendix is available online at `bit.ly/3h1t3lt`.
- The exercises in MyLab Math were improved as follows:
  - We increased the number of assignable algorithmic exercises by about 260. Of these new exercises, about 210 are based on existing textbook exercises. The remaining 50 or so are applications written especially for this revision by the author. These new problems (which do not appear in the book) are labeled EXTRA in MyLab.
  - We modified existing MyLab exercises to improve clarity. These modifications were suggested by the author.
- We greatly increased video coverage, adding 195 new example videos to bring the total to 630. Many of the new videos feature author Rich Evans. Additionally, Rich re-correlated all of the videos to the exercises in MyLab, so when a student chooses "Video" as a learning aid within a MyLab exercise, they can be confident that the video will be helpful.
  - We created a *Guide to Video-Based Assignments*, which makes it easier for you to create assignments containing video by showing which MyLab questions relate to each video.

Continuing Features

PAGE LAYOUT

Special attention has been given to the page layout. We specifically tried to avoid breaking examples or important discussions across pages. Also, all figures are shown immediately adjacent to the material in which they are discussed. Finally, whenever possible, equations or formulas needed for a particular problem are restated rather than referring to formula numbers.

CHAPTER INTRODUCTIONS

Each chapter introduction illustrates specific examples of how the development of technology has been related to the development of mathematics. In these introductions, it is shown that these past discoveries in technology led to some of the methods in mathematics, whereas in other cases mathematical topics already known were later very useful in bringing about advances in technology. Also, each chapter introduction contains a photo that refers to an example that is presented within that chapter.

WORKED-OUT EXAMPLES

EXAMPLE 3 Symbol in capital and in lowercase—forces on a beam

In the study of the forces on a certain beam, the equation $W = \frac{L(wL + 2P)}{8}$ is used. Solve for P.

$$8W = \frac{8L(wL + 2P)}{8} \quad \text{multiply both sides by 8}$$
$$8W = L(wL + 2P) \quad \text{simplify right side}$$
$$8W = wL^2 + 2LP \quad \text{remove parentheses}$$
$$8W - wL^2 = 2LP \quad \text{subtract } wL^2 \text{ from both sides}$$
$$P = \frac{8W - wL^2}{2L} \quad \text{divide both sides by } 2L \text{ and switch sides}$$

- **"Help Text" –** Throughout the book, special explanatory comments in blue type have been used in the examples to emphasize and clarify certain important points. Arrows are often used to indicate clearly the part of the example to which reference is made.
- **Example Descriptions –** A brief descriptive title is given for each example. This gives an easy reference for the example, particularly when reviewing the contents of the section.
- **Application Problems –** There are over 350 applied examples throughout the text that show complete solutions of application problems. Many relate to modern technology such as computer design, electronics, solar energy, lasers, fiber optics, the environment, and space technology. Other examples and exercises relate to technologies such as aeronautics, architecture, automotive, business, chemical, civil, construction, energy, environmental, fire science, machine, medical, meteorology, navigation, police, refrigeration, seismology, and wastewater. The Index of Applications at the end of the book shows the breadth of applications in the text.

KEY FORMULAS AND PROCEDURES

Throughout the book, important formulas are set off and displayed so that they can be easily referenced for use. Similarly, summaries of techniques and procedures consistently appear in color-shaded boxes. The back endpapers of the text provide a handy reference of key formulas and facts.

"CAUTION" AND "NOTE" INDICATORS

CAUTION This heading is used to identify errors students commonly make or places where they frequently have difficulty. ■

NOTE ▸ The NOTE label in the side margin, along with [accompanying blue brackets in the main body of the text,] points out material that is of particular importance in developing or understanding the topic under discussion.

GRAPHING CALCULATOR HANDBOOK

The margins of the text contain short URLs that take students directly to the relevant content in the Graphing Calculator Handbook (which was written by Benjamin Rushing of Northwestern State University). If you'd like to see a complete listing of entries for the online graphing calculator handbook, go to `bit.ly/2NFYzDK`.

CHAPTER AND SECTION CONTENTS

A listing of learning outcomes for each chapter is given on the introductory page of the chapter. Also, a listing of the key topics of each section is given below the section number and title on the first page of the section. This gives the student and instructor a quick preview of the chapter and section contents.

PRACTICE EXERCISES

Most sections include some practice exercises in the margin. They are included so that a student is more actively involved in the learning process and can check his or her understanding of the material. They can also be used for classroom exercises. The answers to these exercises are given at the end of the exercises set for the section. There are over 450 of these exercises.

FEATURES OF EXERCISES

- **Exercises Directly Referenced to Text Examples –** The first few exercises in most of the text sections are referenced directly to a specific example of the section. These exercises are worded so that it is necessary for the student to refer to the example in order to complete the required solution. In this way, the student should be able to better review and understand the text material before attempting to solve the exercises that follow.
- **Writing Exercises –** There are over 270 writing exercises through the book (at least eight in each chapter) that require at least a sentence or two of explanation as part of the answer. These are noted by a pencil icon next to the exercise number.
- **Application Problems –** There are about 3000 application exercises in the text that represent the breadth of applications that students will encounter in their chosen professions. The Index of Applications at the end of the book shows the breadth of applications in the text.

CHAPTER ENDMATTER

- **Key Formulas & Equations –** Here, all important formulas and equations are listed together with their corresponding equation numbers for easy reference.
- **Chapter Review Exercises –** These exercises consist of (a) Concept Check Exercises (a set of true/false exercises) and (b) Practice and Applications.
- **Chapter Test –** These are designed to mirror what students might see on the actual chapter test. Complete step-by-step solutions to all practice test problems are given in the back of the book.

MARGIN NOTES

Throughout the text, some margin notes point out relevant historical events in mathematics and technology. Other margin notes are used to make specific comments related to the text material. Also, where appropriate, equations from earlier material are shown for reference in the margin.

ANSWERS TO EXERCISES

The answers to odd-numbered exercises are given near the end of the book. The Student's Solution Manual contains solutions to every other odd-numbered exercise and the Instructor's Solution Manual contains solutions to all section exercises.

FLEXIBILITY OF COVERAGE

The order of coverage can be changed in many places and certain sections may be omitted without loss of continuity of coverage. Users of earlier editions have indicated successful use of numerous variations in coverage. Any changes will depend on the type of course and completeness required.

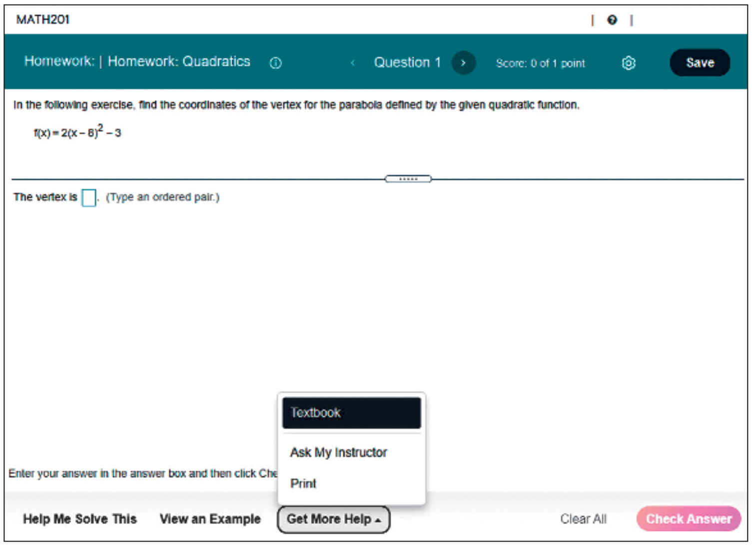
MyLab™ Math Resources for Success

MyLab Math (pearson.com/mylab/math) is available to accompany Pearson's market-leading text options, including this text (access code required). MyLab Math is the teaching and learning platform that empowers you to reach every student. It combines trusted author content—including full eText and online homework with immediate feedback—with digital tools and a flexible platform to personalize the learning experience and improve results for each student.

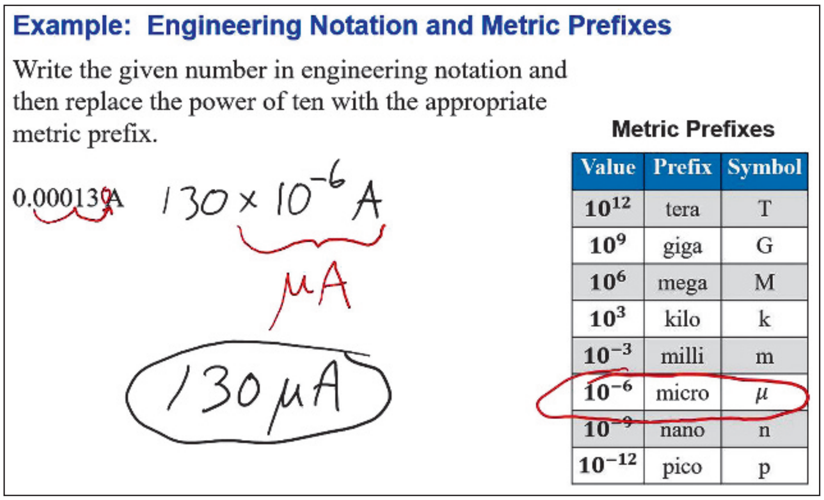
MYLAB MATH STUDENT RESOURCES

Each student learns at a different pace. Personalized learning pinpoints the precise areas where each student needs practice, giving all students the support they need — when and where they need it — to be successful.

- **Exercises with Immediate Feedback –** The 3650 exercises in MyLab Math (260 of them new for this edition) reflect the approach and learning style of this text, and regenerate algorithmically to give students unlimited opportunity for practice and mastery. Most exercises include learning aids, such as guided solutions, videos, and sample problems, and they offer helpful feedback when students enter incorrect answers.



- **Instructional videos** – The 630 videos (195 of them new for this edition) in the 12th edition MyLab Math course provide help for students outside of the classroom. These videos are also available as learning aids within the homework exercises, for students to refer to at point-of-use.



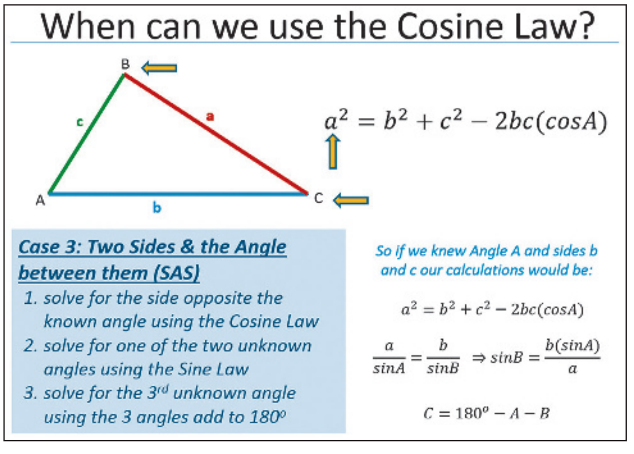
- **The complete eText** is available to students through their MyLab Math course. The eText includes links to videos. The eText is also available for stand-alone purchase.
- **Online Graphing Calculator Handbook**, created specifically for this text by Benjamin Rushing (Northwestern State University), features instructions for the TI-84 and TI-89 family of calculators. Links to specific parts of the handbook are included as short URLs that appear throughout the textbook. If you'd like to see a complete listing of entries for the online graphing calculator handbook, go to `bit.ly/2NFYzDK`.
- **Mindset videos** and corresponding assignable, open-ended exercises foster a growth mindset in students. This material encourages them to maintain a positive attitude about learning, value their own ability to grow, and view mistakes as learning opportunities — so often a hurdle for math students.

- **Personal Inventory Assessments** are a collection of online exercises designed to promote self-reflection and engagement in students. These 33 assessments include topics such as a Stress Management Assessment, Diagnosing Poor Performance and Enhancing Motivation, and Time Management Assessment.
- **Student Solutions Manual** – Contains fully worked solutions to odd-numbered exercises. Available for download from within MyLab Math.

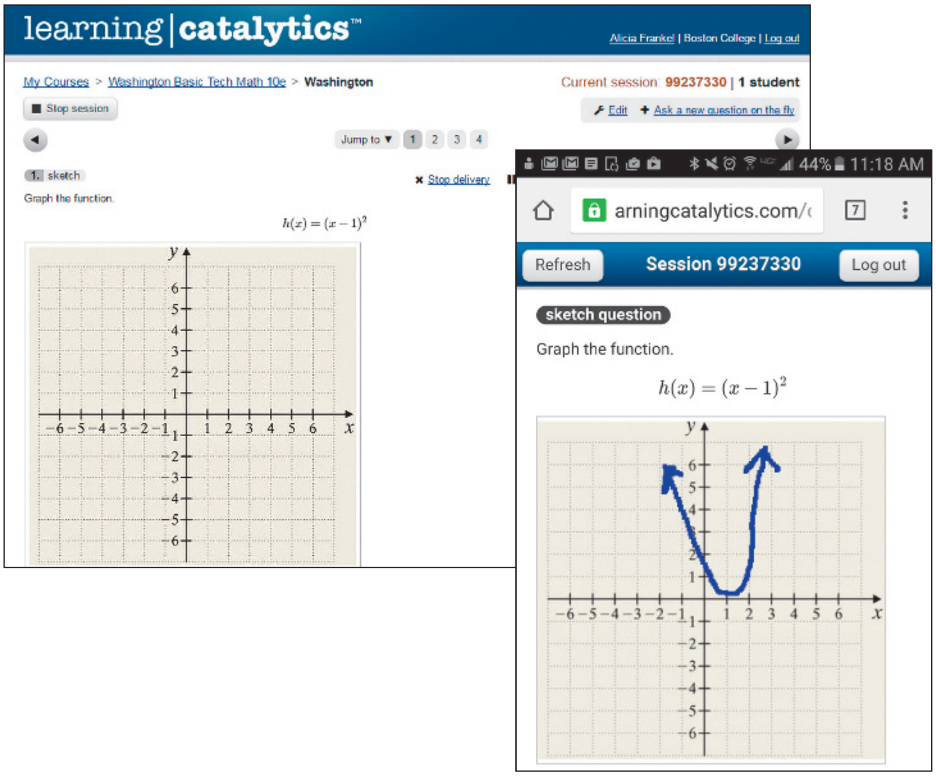
MYLAB MATH INSTRUCTOR RESOURCES

Your course is unique. Whether you'd like to build your own assignments, teach multiple sections, or set prerequisites, MyLab gives you the flexibility to easily create your course to fit your needs.

- **PowerPoint®** files feature animations that are designed to help you better teach key concepts.



- **Learning Catalytics** – With Learning Catalytics™, you'll hear from every student when it matters most. You pose a variety of questions in class (choosing from pre-loaded questions or questions of your own making) that help students recall ideas, apply concepts, and develop critical-thinking skills. Your students respond using their own smartphones, tablets, or laptops.



- **Guide to Video-Based Assignments** – Makes it easier for you to create assignments containing video by showing which MyLab questions relate to each video.
- **Instructor's Solution Manual** by Matthew Hudelson (Washington State University) contains detailed solutions to every section exercise, including review exercises.
- **Accessibility** – Throughout our development process for every release, we test and retest the capabilities of our products against the highest standards. Many existing products meet the Web Content Accessibility Guidelines (WCAG) 2.0 guidelines. As of Summer 2020, new product development has worked to meet the WCAG 2.1 AA guidelines. More information can be found at pearson.com/us/accessibility.
- **TestGen** enables instructors to build, edit, print, and administer tests using a bank of questions developed to cover all objectives in the text. TestGen is algorithmically based, allowing you to create multiple but equivalent versions of the same question or test. Instructors can also modify test bank questions or add new questions. The TestGen software and accompanying test bank are available to qualified instructors for download from Pearson's online catalog.

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- Scott Zhang, Conestoga College
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Finally, I wish to sincerely thank again each of the over 410 reviewers of the twelve editions of this text. Their comments have helped further the education of more than two million students since this text was first published in 1964.

Rich Evans

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Basic Algebraic Operations

1

Interest in things such as the land on which they lived, the structures they built, and the motion of the planets led people in early civilizations to keep records and to create methods of counting and measuring.

In turn, some of the early ideas of arithmetic, geometry, and trigonometry were developed. From such beginnings, mathematics has played a key role in the great advances in science and technology.

Often, mathematical methods were developed from scientific studies made in particular areas, such as astronomy and physics. Many people were interested in the math itself and added to what was then known. Although this additional mathematical knowledge may not have been related to applications at the time it was developed, it often later became useful in applied areas.

In the chapter introductions that follow, examples of the interaction of technology and mathematics are given. From these examples and the text material, it is hoped you will better understand the important role that math has had and still has in technology. In this text, there are applications from technologies including (but not limited to) aeronautical, business, communications, electricity, electronics, engineering, environmental, heat and air conditioning, mechanical, medical, meteorology, petroleum, product design, solar, and space.

We begin by reviewing the concepts that deal with numbers and symbols. This will enable us to develop topics in algebra, an understanding of which is essential for progress in other areas such as geometry, trigonometry, and calculus.

LEARNING OUTCOMES

After completion of this chapter, the student should be able to:

- Identify real, imaginary, rational, and irrational numbers
- Perform mathematical operations on integers, decimals, fractions, and radicals
- Use the fundamental laws of algebra in numeric and algebraic expressions
- Employ mathematical order of operations
- Understand technical measurement, approximation, the use of significant digits, and rounding
- Use scientific and engineering notations
- Convert units of measurement
- Rearrange and solve basic algebraic equations
- Interpret word problems using algebraic symbols

◀ From the Great Pyramid of Giza, built in Egypt 4500 years ago, to the modern technology of today, mathematics has played a key role in the advancement of civilization. Along the way, important discoveries have been made in areas such as architecture, navigation, transportation, electronics, communication, and astronomy. Mathematics will continue to pave the way for new discoveries.

1.1 Numbers

Real Number System • Number Line • Absolute Value • Signs of Inequality • Reciprocal • Denominate Numbers • Literal Numbers

In technology and science, as well as in everyday life, we use the very familiar **counting numbers,** or **natural numbers** 1, 2, 3, and so on. The **whole numbers** include 0 as well as all the natural numbers. Because it is necessary and useful to use negative numbers as well as positive numbers in mathematics and its applications, the natural numbers are called the **positive integers**, and the numbers -1, -2, -3, and so on are the **negative integers.**

Therefore, *the* **integers** *include the positive integers, the negative integers*, and **zero,** *which is neither positive nor negative*. This means that the integers are the numbers . . . , $-3, -2, -1, 0, 1, 2, 3, \ldots$ and so on.

A **rational number** *is a number that can be expressed as the division of one integer **a** by another nonzero integer b, and can be represented by the fraction **a**/**b**.* Here ***a*** *is the* **numerator** *and* ***b*** *is the* **denominator.** Here we have used algebra by letting letters represent numbers.

■ Irrational numbers were discussed by the Greek mathematician Pythagoras in about 540 B.C.E.

Another type of number, an **irrational number,** *cannot be written in the form of a fraction that is the division of one integer by another integer*. The following example illustrates integers, rational numbers, and irrational numbers.

EXAMPLE 1 Identifying rational numbers and irrational numbers

The numbers 5 and -19 are integers. They are also rational numbers because they can be written as $\frac{5}{1}$ and $\frac{-19}{1}$, respectively. Normally, we do not write the 1's in the denominators.

■ For reference, $\pi = 3.14159265\ldots$

The numbers $\frac{5}{8}$ and $\frac{-11}{3}$ are rational numbers because the numerator and the denominator of each are integers.

The numbers $\sqrt{2}$ and π are irrational numbers. It is not possible to find two integers, one divided by the other, to represent either of these numbers. In decimal form, irrational numbers are nonterminating, nonrepeating decimals. It can be shown that square roots (and other roots) that cannot be expressed exactly in decimal form are irrational. Also, $\frac{22}{7}$ is sometimes used as an *approximation* for π, but it is not equal *exactly* to π. We must remember that $\frac{22}{7}$ is rational and π is irrational.

■ A notation that is often used for repeating decimals is to place a bar over the digits that repeat. Using this notation we can write $\frac{1121}{1665} = 0.6\overline{732}$ and $\frac{2}{3} = 0.\overline{6}$.

The decimal number 1.5 is rational since it can be written as $\frac{3}{2}$. Any such *terminating decimal* is rational. The number $0.6666\ldots$, where the 6's continue on indefinitely, is rational because we may write it as $\frac{2}{3}$. In fact, any *repeating decimal* (in decimal form, a specific sequence of digits is repeated indefinitely) is rational. The decimal number $0.6732732732\ldots$ is a repeating decimal where the sequence of digits 732 is repeated indefinitely $\left(0.6732732732\ldots = \frac{1121}{1665}\right)$. ■

The rational numbers together with the irrational numbers, including all such numbers that are positive, negative, or zero, make up the **real number system** (see Fig. 1.1). There are times we will encounter an **imaginary number,** *the name given to the square root of a*

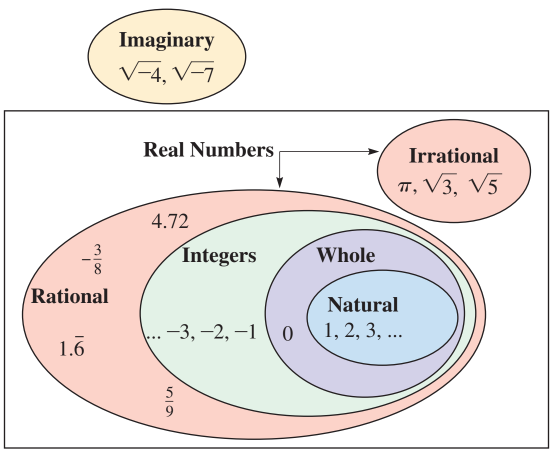

Fig. 1.1

negative number. Imaginary numbers are not real numbers and will be discussed in Chapter 12. However, unless specifically noted, we will use real numbers. Until Chapter 12, it will be necessary to only *recognize* imaginary numbers when they occur.

Also in Chapter 12, we will consider **complex numbers,** which include both the real numbers and imaginary numbers. See Exercise 39 of this section.

■ Real numbers and imaginary numbers are both included in the *complex number system*. See Exercise 39.

EXAMPLE 2 Identifying real numbers and imaginary numbers

(a) The number 7 is an integer. It is also rational because $7 = \frac{7}{1}$, and it is a real number since the real numbers include all the rational numbers.

(b) The number 3π is irrational, and it is real because the real numbers include all the irrational numbers.

(c) The numbers $\sqrt{-10}$ and $-\sqrt{-7}$ are imaginary numbers.

(d) The number $\frac{-3}{7}$ is rational and real. The number $-\sqrt{7}$ is irrational and real.

(e) The number $\frac{\pi}{6}$ is irrational and real. The number $\frac{\sqrt{-3}}{2}$ is imaginary. ■

■ Fractions were used by early Egyptians and Babylonians. They were used for calculations that involved parts of measurements, property, and possessions.

A **fraction** *may contain any number or symbol representing a number in its numerator or in its denominator*. The fraction indicates the division of the numerator by the denominator, as we previously indicated in writing rational numbers. Therefore, a fraction may be a number that is rational, irrational, or imaginary.

EXAMPLE 3 Fractions

(a) The numbers $\frac{2}{7}$ and $\frac{-3}{2}$ are fractions, and they are rational.

(b) The numbers $\frac{\sqrt{2}}{9}$ and $\frac{6}{\pi}$ are fractions, but they are not rational numbers. It is not possible to express either as one integer divided by another integer.

(c) The number $\frac{\sqrt{-5}}{6}$ is a fraction, and it is an imaginary number. ■

THE NUMBER LINE

Real numbers may be represented by points on a line. We draw a horizontal line and designate some point on it by O, which we call the **origin** (see Fig. 1.2). The integer *zero* is located at this point. Equal intervals are marked to the right of the origin, and the positive integers are placed at these positions. The other positive rational numbers are located between the integers. The points that cannot be defined as rational numbers represent irrational numbers. We cannot tell whether a given point represents a rational number or an irrational number unless it is specifically marked to indicate its value.

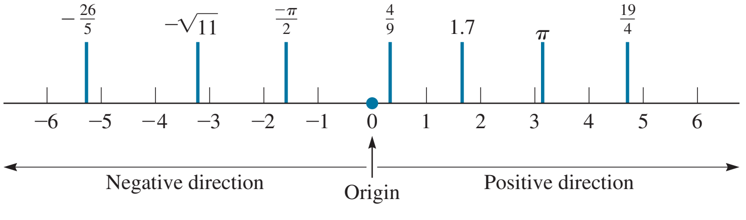

Fig. 1.2

The negative numbers are located on the number line by starting at the origin and marking off equal intervals *to the left, which is the* **negative direction.** As shown in Fig. 1.2, *the positive numbers are to the right of the origin and the negative numbers are to the left of the origin*. Representing numbers in this way is especially useful for graphical methods.

We next define another important concept of a number. *The* **absolute value** *of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number*. On the number line, we may interpret the absolute value of a number as the distance (which is always positive) between the origin and the number. Absolute value is denoted by writing the number between vertical lines, as shown in the following example.

EXAMPLE 4 Absolute value

The absolute value of 6 is 6, and the absolute value of -7 is 7. We write these as $|6| = 6$ and $|-7| = 7$. See Fig. 1.3.

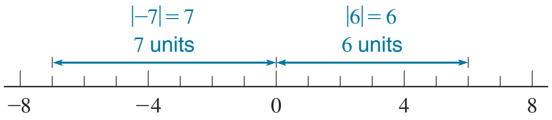

Fig. 1.3

Practice Exercises

1. $|-4.2| = ?$ **2.** $-\left|-\frac{3}{4}\right| = ?$

Other examples are $|\frac{7}{5}| = \frac{7}{5}$, $|-\sqrt{2}| = \sqrt{2}$, $|0| = 0$, $-|\pi| = -\pi$, $|-5.29| = 5.29$, and $-|-9| = -9$ since $|-9| = 9$. ■

■ The symbols $=$, $<$, and $>$ were introduced by English mathematicians in the late 1500s.

On the number line, *if a first number is to the right of a second number, then the first number is said to be* **greater than** *the second. If the first number is to the left of the second, it is* **less than** *the second number*. The symbol $>$ designates "is greater than," and the symbol $<$ designates "is less than." These are called **signs of inequality.** See Fig. 1.4.

EXAMPLE 5 Signs of inequality

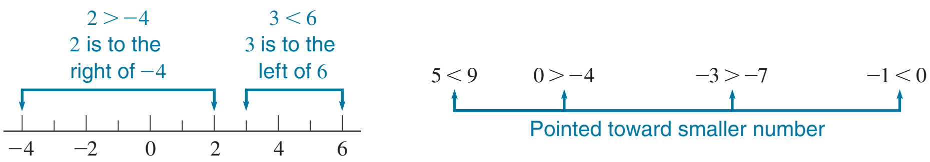

Fig. 1.4 ■

Practice Exercises

Place the correct sign of inequality ($<$ or $>$) between the given numbers.

3. $-5 \quad 4$ **4.** $0 \quad -3$

Every number, except zero, has a **reciprocal.** *The reciprocal of a number is 1 divided by the number*.

EXAMPLE 6 Reciprocal

The reciprocal of 7 is $\frac{1}{7}$. The reciprocal of $\frac{2}{3}$ is

$$\frac{1}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2} \quad \text{invert denominator and multiply (from arithmetic)}$$

The reciprocal of 0.5 is $\frac{1}{0.5} = 2$. The reciprocal of $-\pi$ is $-\frac{1}{\pi}$. Note that the negative sign is retained in the reciprocal of a negative number.

We showed the multiplication of 1 and $\frac{3}{2}$ as $1 \times \frac{3}{2}$. We could also show it as $1 \cdot \frac{3}{2}$ or $1(\frac{3}{2})$. We will often find the form with parentheses is preferable. ■

Practice Exercise

5. Find the reciprocals of

(a) -4 **(b)** $\frac{3}{8}$

In applications, *numbers that represent a measurement and are written with units of measurement are called* **denominate numbers.** The next example illustrates the use of units and the symbols that represent them.

■ For reference, see Appendix B for units of measurement and the symbols used for them.

EXAMPLE 7 Denominate numbers

(a) To show that a certain TV weighs 62 pounds, we write the weight as 62 lb.

(b) To show that a giant redwood tree is 330 feet high, we write the height as 330 ft.

(c) To show that the speed of a rocket is 1500 meters per second, we write the speed as 1500 m/s. (Note the use of s for second. We use s rather than sec.)

(d) To show that the area of a computer chip is 0.75 square inch, we write the area as 0.75 in.^{2}. (We will not use sq in.)

(e) To show that the volume of water in a glass tube is 25 cubic centimeters, we write the volume as 25 cm^3. (We will not use cu cm nor cc.) ■

It is usually more convenient to state definitions and operations on numbers in a general form. *To do this, we represent the numbers by letters, called* **literal numbers.** For example, if we want to say "If a first number is to the right of a second number on the number line, then the first number is greater than the second number," we can write "If a is to the right of b on the number line, then $a > b$." Another example of using a literal number is "The reciprocal of n is $1/n$."

Certain literal numbers may take on any allowable value, whereas other literal numbers represent the same value throughout the discussion. *Those literal numbers that may vary in a given problem are called* **variables,** and *those literal numbers that are held fixed are called* **constants.**

EXAMPLE 8 Variables and constants

(a) The resistance of an electric resistor is R. The current I in the resistor equals the voltage V divided by R, written as $I = V/R$. For this resistor, I and V may take on various values, and R is fixed. This means I and V are variables and R is a constant. For a *different* resistor, the value of R may differ.

(b) The fixed cost for a calculator manufacturer to operate a certain plant is b dollars per day, and it costs a dollars to produce each calculator. The total daily cost C to produce n calculators is

$$C = an + b$$

Here, C and n are variables, and a and b are constants, and the product of a and n is shown as an. For *another* plant, the values of a and b would probably differ.

If specific numerical values of a and b are known, say $a = \$7$ per calculator and $b = \$3000$, then $C = 7n + 3000$. Thus, constants may be numerical or literal. ■

EXERCISES 1.1

In Exercises 1–4, make the given changes in the indicated examples of this section, and then answer the given questions.

1. In the first line of Example 1, change the 5 to -7 and the -19 to 12. What other changes must then be made in the first paragraph?
2. In Example 4, change the 6 to -6. What other changes must then be made in the first paragraph?
3. In the left figure of Example 5, change the 2 to -6. What other changes must then be made?
4. In Example 6, change the $\frac{2}{3}$ to $\frac{3}{2}$. What other changes must then be made?

In Exercises 5–8, designate each of the given numbers as being an integer, rational, irrational, real, or imaginary. (More than one designation may be correct.)

5. $3, \sqrt{-4}$ **6.** $\frac{\sqrt{7}}{3}, -6$ **7.** $-\frac{\pi}{6}, \frac{1}{8}$ **8.** $-\sqrt{-6}, -2.33$

In Exercises 9 and 10, find the absolute value of each real number.

9. $3, -3, \frac{\pi}{4}, \sqrt{-1}$ **10.** $-0.857, \sqrt{2}, -\frac{19}{4}, \frac{\sqrt{-5}}{-2}$

In Exercises 11–18, insert the correct sign of inequality (> or <) between the given numbers.

11. $6 \quad 8$ **12.** $7 \quad 5$

13. $\pi \quad 3.1416$ **14.** $-4 \quad 0$

15. $-4 \quad -|-3|$ **16.** $-\sqrt{2} \quad -1.42$

17. $-\frac{2}{3} \quad -\frac{3}{4}$ **18.** $-0.6 \quad 0.2$

In Exercises 19 and 20, find the reciprocal of each number.

19. $3, -\frac{4}{\sqrt{3}}, \frac{y}{b}$ **20.** $-\frac{1}{3}, 0.25, 2x$

In Exercises 21 and 22, locate (approximately) each number on a number line as in Fig. 1.2.

21. $2.5, -\frac{12}{5}, \sqrt{3}, -\frac{3}{4}$ **22.** $-\frac{\sqrt{2}}{2}, 2\pi, \frac{123}{19}, -\frac{7}{3}$

In Exercises 23–46, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

23. Is an absolute value always positive? Explain.

24. Is -2.17 rational? Explain.

25. What is the reciprocal of the reciprocal of any positive or negative number?

26. Is the repeating decimal $2.\overline{72}$ rational or irrational?

27. True or False: A nonterminating, nonrepeating decimal is an irrational number.

28. If $b > a$ and $a > 0$, is $|b - a| < |b| - |a|$?

29. List the following numbers in numerical order, starting with the smallest: $-1, 9, \pi, \sqrt{5}, |-8|, -|-3|, -3.1$.

30. List the following numbers in numerical order, starting with the smallest: $\frac{1}{5}, -\sqrt{10}, -|-6|, -4, 0.25, |-\pi|$.

31. If a and b are positive integers and $b > a$, what type of number is represented by the following?

(a) $b - a$ **(b)** $a - b$ **(c)** $\frac{b - a}{b + a}$

32. If a and b represent positive integers, what kind of number is represented by (a) $a + b$, (b) a/b, and (c) $a \times b$?

33. For any positive or negative integer: (a) Is its absolute value always an integer? (b) Is its reciprocal always a rational number?

34. For any positive or negative rational number: (a) Is its absolute value always a rational number? (b) Is its reciprocal always a rational number?

35. Describe the location of a number x on the number line when (a) $x > 0$ and (b) $x < -4$.

36. Describe the location of a number x on the number line when (a) $|x| < 1$ and (b) $|x| > 2$.

37. For a number $x > 1$, describe the location on the number line of the reciprocal of x.

38. For a number $x < 0$, describe the location on the number line of the number with a value of $|x|$.

39. A *complex number* is defined as $a + bj$, where a and b are real numbers and $j = \sqrt{-1}$. For what values of a and b is the complex number $a + bj$ a real number? (All real numbers and all imaginary numbers are also complex numbers.)

40. A sensitive gauge measures the total weight w of a container and the water that forms in it as vapor condenses. It is found that $w = c\sqrt{0.1t + 1}$, where c is the weight of the container and t is the time of condensation. Identify the variables and constants.

41. In an electric circuit, the reciprocal of the total capacitance of two capacitors in series is the sum of the reciprocals of the capacitances $\left(\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}\right)$. Find the total capacitance of two capacitances of 0.0040 F and 0.0010 F connected in series.

42. Alternating-current (ac) voltages change rapidly between positive and negative values. If a voltage of 100 V changes to -200 V, which is greater in absolute value?

43. The memory of a certain computer has a bits in each byte. Express the number N of bits in n kilobytes in an equation. (A *bit* is a single digit, and bits are grouped in *bytes* in order to represent special characters. Generally, there are 8 bits per byte. If necessary, see Appendix B for the meaning of *kilo*.)

44. The computer design of the base of a truss is x ft long. Later it is redesigned and shortened by y in. Give an equation for the length L, in inches, of the base in the second design.

45. In a laboratory report, a student wrote "$-20°C > -30°C$." Is this statement correct? Explain.

46. After 5 s, the pressure on a valve is less than 60 lb/in.^{2} (pounds per square inch). Using t to represent time and p to represent pressure, this statement can be written "for $t > 5$ s, $p < 60$ lb/in.^{2}." In this way, write the statement "when the current I in a circuit is less than 4 A, the resistance R is greater than 12 Ω (ohms)."

Answers to Practice Exercises

1. 4.2 **2.** $-\frac{3}{4}$ **3.** $<$ **4.** $>$ **5.** **(a)** $-\frac{1}{4}$ **(b)** $\frac{8}{3}$

1.2 Fundamental Operations of Algebra

Fundamental Laws of Algebra • Operations on Positive and Negative Numbers • Order of Operations • Operations with Zero

If two numbers are added, it does not matter in which order they are added. (For example, $5 + 3 = 8$ and $3 + 5 = 8$, or $5 + 3 = 3 + 5$.) This statement, generalized and accepted as being correct for all possible combinations of numbers being added, is called the **commutative law** for addition. It states that *the sum of two numbers is the same,* *regardless of the order in which they are added*. We make no attempt to prove this law in general, but accept that it is true.

In the same way, we have the **associative law** for addition, which states that *the sum of three or more numbers is the same, regardless of the way in which they are grouped for addition*. For example, $3 + (5 + 6) = (3 + 5) + 6$.

The laws just stated for addition are also true for multiplication. Therefore, *the product of two numbers is the same, regardless of the order in which they are multiplied,* and *the product of three or more numbers is the same, regardless of the way in which they are grouped for multiplication*. For example, $2 \times 5 = 5 \times 2$, and $5 \times (4 \times 2) = (5 \times 4) \times 2$.

Another very important law is the **distributive law.** It states that *the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum*. For example,

■ Note carefully the difference:
associative law: $5 \times (4 \times 2)$
distributive law: $5 \times (4 + 2)$

$$5(4 + 2) = 5 \times 4 + 5 \times 2$$

In this case, it can be seen that the total is 30 on each side.

In practice, we use these **fundamental laws of algebra** naturally without thinking about them, except perhaps for the distributive law.

Not all operations are commutative and associative. For example, division is not commutative, because the order of division of two numbers does matter. For instance, $\frac{6}{5} \neq \frac{5}{6}$ ($\neq$ is read "does not equal"). (Also, see Exercise 54.)

Using literal numbers, the fundamental laws of algebra are as follows:

Commutative law of addition: $a + b = b + a$

Associative law of addition: $a + (b + c) = (a + b) + c$

Commutative law of multiplication: $ab = ba$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

■ Note the meaning of *identity*.

Each of these laws is an example of an *identity,* in that the expression to the left of the $=$ sign equals the expression to the right for any value of each of *a, b,* and *c*.

OPERATIONS ON POSITIVE AND NEGATIVE NUMBERS

When using the basic operations (addition, subtraction, multiplication, division) on positive and negative numbers, we determine the result to be either positive or negative according to the following rules.

Addition of two numbers of the same sign *Add their absolute values and assign the sum their common sign.*

EXAMPLE 1 Adding numbers of the same sign

(a) $2 + 6 = 8$ — the sum of two positive numbers is positive

(b) $-2 + (-6) = -(2 + 6) = -8$ — the sum of two negative numbers is negative

■ From Section 1.1, we recall that a positive number is preceded by no sign. Therefore, in using these rules, we show the "sign" of a positive number by simply writing the number itself.

The negative number -6 is placed in parentheses because it is also preceded by a plus sign showing addition. It is not necessary to place the -2 in parentheses. ■

Addition of two numbers of different signs *Subtract the number of smaller absolute value from the number of larger absolute value and assign to the result the sign of the number of larger absolute value.*

EXAMPLE 2 Adding numbers of different signs

(a) $2 + (-6) = -(6 - 2) = -4$ — the negative 6 has the larger absolute value
(b) $-6 + 2 = -(6 - 2) = -4$
(c) $6 + (-2) = 6 - 2 = 4$ — the positive 6 has the larger absolute value
(d) $-2 + 6 = 6 - 2 = 4$

the subtraction of absolute values ■

Subtraction of one number from another *Change the sign of the number being subtracted and change the subtraction to addition. Perform the addition.*

EXAMPLE 3 Subtracting positive and negative numbers

(a) $2 - 6 = 2 + (-6) = -(6 - 2) = -4$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 2(a).

(b) $-2 - 6 = -2 + (-6) = -(2 + 6) = -8$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 1(b).

(c) $-a - (-a) = -a + a = 0$

NOTE ▸ This shows that subtracting a number from itself results in zero, even if the number is negative. [Subtracting a negative number is equivalent to adding a positive number of the same absolute value.]

(d) $-2 - (-6) = -2 + 6 = 4$

(e) The change in temperature from $-12°C$ to $-26°C$ is

$-26°C - (-12°C) = -26°C + 12°C = -14°C$ ■

Multiplication and division of two numbers *The product (or quotient) of two numbers of the same sign is positive. The product (or quotient) of two numbers of different signs is negative.*

EXAMPLE 4 Multiplying and dividing positive and negative numbers

(a) $3(12) = 3 \times 12 = 36$ $\quad \frac{12}{3} = 4$ — result is positive if both numbers are positive

(b) $-3(-12) = 3 \times 12 = 36$ $\quad \frac{-12}{-3} = 4$ — result is positive if both numbers are negative

(c) $3(-12) = -(3 \times 12) = -36$ $\quad \frac{-12}{3} = -\frac{12}{3} = -4$ — result is negative if one number is positive and the other is negative

(d) $-3(12) = -(3 \times 12) = -36$ $\quad \frac{12}{-3} = -\frac{12}{3} = -4$ ■

Practice Exercises

Evaluate: **1.** $-5 - (-8)$ **2.** $-5(-8)$

ORDER OF OPERATIONS

Often, how we are to combine numbers is clear by grouping the numbers using symbols such as **parentheses,** ( ); the **bar,** ____, between the numerator and denominator of a fraction; and **vertical lines** for absolute value. Otherwise, for an expression in which there are several operations, we use the following order of operations.

Order of Operations

1. Perform operations within grouping symbols (parentheses, brackets, or absolute value symbols).
2. Perform multiplications and divisions (from left to right).
3. Perform additions and subtractions (from left to right).

EXAMPLE 5 Order of operations

■ Note that $20 \div (2 + 3) = \frac{20}{2+3}$, whereas $20 \div 2 + 3 = \frac{20}{2} + 3$.

(a) $20 \div (2 + 3)$ is evaluated by first adding $2 + 3$ and then dividing. The grouping of $2 + 3$ is clearly shown by the parentheses. Therefore, $20 \div (2 + 3) = 20 \div 5 = 4$.

(b) $20 \div 2 + 3$ is evaluated by first dividing 20 by 2 and then adding. No specific grouping is shown, and therefore the division is done before the addition. This means $20 \div 2 + 3 = 10 + 3 = 13$.

NOTE ➧ **(c)** [$16 - 2 \times 3$ is evaluated by *first multiplying* 2 by 3 and then subtracting. We do not first subtract 2 from 16.] Therefore, $16 - 2 \times 3 = 16 - 6 = 10$.

(d) $16 \div 2 \times 4$ is evaluated by first dividing 16 by 2 and then multiplying. From left to right, the division occurs first. Therefore, $16 \div 2 \times 4 = 8 \times 4 = 32$.

(e) $|3 - 5| - |-3 - 6|$ is evaluated by first performing the subtractions within the absolute value vertical bars, then evaluating the absolute values, and then subtracting. This means that $|3 - 5| - |-3 - 6| = |-2| - |-9| = 2 - 9 = -7$. ■

Practice Exercises

Evaluate: **3.** $12 - 6 \div 2$

4. $16 \div (2 \times 4)$

When evaluating expressions, it is generally more convenient to change the operations and numbers so that the result is found by the addition and subtraction of positive numbers. When this is done, we must remember that

$$a + (-b) = a - b \tag{1.1}$$

$$a - (-b) = a + b \tag{1.2}$$

EXAMPLE 6 Evaluating numerical expressions

(a) $7 + (-3) - 6 = 7 - 3 - 6 = 4 - 6 = -2$ using Eq. (1.1)

(b) $\frac{18}{-6} + 5 - (-2)(3) = -3 + 5 - (-6) = 2 + 6 = 8$ using Eq. (1.2)

(c) $\frac{|3 - 15|}{-2} - \frac{8}{4 - 6} = \frac{12}{-2} - \frac{8}{-2} = -6 - (-4) = -6 + 4 = -2$

(d) $\frac{-12}{2 - 8} + \frac{5 - 1}{2(-1)} = \frac{-12}{-6} + \frac{4}{-2} = 2 + (-2) = 2 - 2 = 0$

In illustration (b), we see that the division and multiplication were done before the addition and subtraction. In (c) and (d), we see that the groupings were evaluated first. Then we did the divisions, and finally the subtraction and addition. ■

Practice Exercises

Evaluate: **5.** $2(-3) - \frac{4 - 8}{2}$

6. $\frac{|5 - 15|}{2} - \frac{-9}{3}$

EXAMPLE 7 Evaluating—velocity after collision

A 3000-lb van going at 40 mi/h ran head-on into a 2000-lb car going at 20 mi/h. An insurance investigator determined the velocity of the vehicles immediately after the collision from the following calculation. See Fig. 1.5.

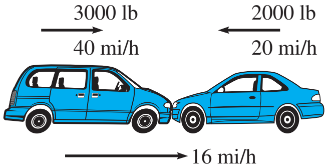

Fig. 1.5

$$\frac{3000(40) + (2000)(-20)}{3000 + 2000} = \frac{120{,}000 + (-40{,}000)}{3000 + 2000} = \frac{120{,}000 - 40{,}000}{5000}$$

$$= \frac{80{,}000}{5000} = 16 \text{ mi/h}$$

The numerator and the denominator must be evaluated before the division is performed. The multiplications in the numerator are performed first, followed by the addition in the denominator and the subtraction in the numerator. ■