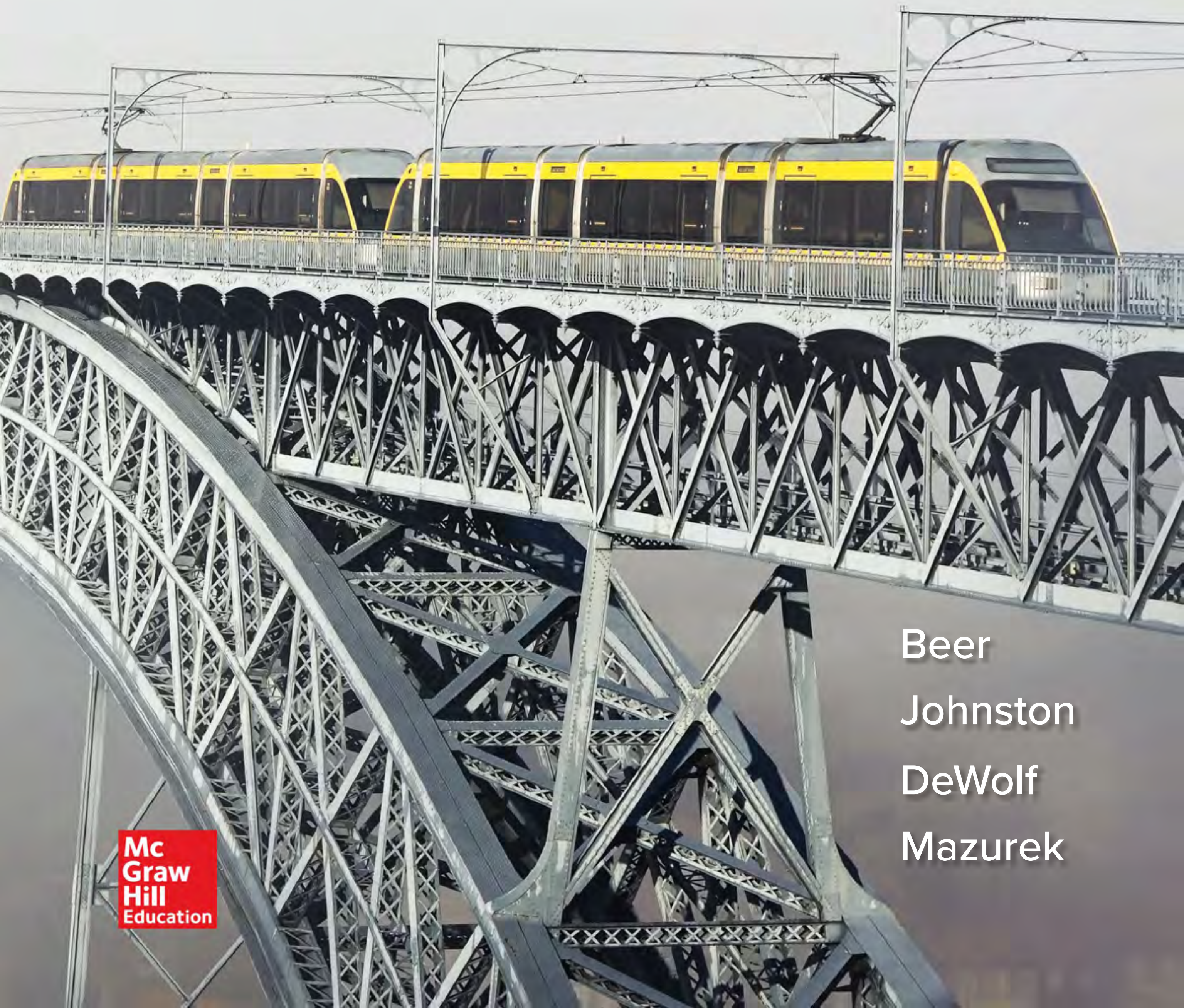
Eighth Edition

MECHANICS of MATERIALS



Beer
Johnston
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Eighth Edition

Mechanics of Materials

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Contents

Preface vii
Guided Tour xii
List of Symbols xiv

1 Introduction—Concept of Stress 3

1.1 Review of The Methods of Statics 4
1.2 Stresses in the Members of a Structure 7
1.3 Stress on an Oblique Plane Under Axial Loading 27
1.4 Stress Under General Loading Conditions; Components of Stress 28
1.5 Design Considerations 31
Review and Summary 45

2 Stress and Strain—Axial Loading 57

2.1 An Introduction to Stress and Strain 59
2.2 Statically Indeterminate Problems 80
2.3 Problems Involving Temperature Changes 84
2.4 Poisson's Ratio 96
2.5 Multiaxial Loading: Generalized Hooke's Law 97
*2.6 Dilatation and Bulk Modulus 99
2.7 Shearing Strain 101
2.8 Deformations Under Axial Loading—Relation Between E, ν, and G 104
*2.9 Stress-Strain Relationships for Fiber-Reinforced Composite Materials 106
2.10 Stress and Strain Distribution Under Axial Loading: Saint-Venant's Principle 117
2.11 Stress Concentrations 119
2.12 Plastic Deformations 121
*2.13 Residual Stresses 124
Review and Summary 135

3 Torsion 149

3.1 Circular Shafts in Torsion 152
3.2 Angle of Twist in the Elastic Range 168
3.3 Statically Indeterminate Shafts 171
3.4 Design of Transmission Shafts 186
3.5 Stress Concentrations in Circular Shafts 188
*3.6 Plastic Deformations in Circular Shafts 196
*3.7 Circular Shafts Made of an Elastoplastic Material 197
*3.8 Residual Stresses in Circular Shafts 200
*3.9 Torsion of Noncircular Members 210
*3.10 Thin-Walled Hollow Shafts 212
Review and Summary 224

*Advanced or specialty topics

4 Pure Bending 237

4.1 Symmetric Members in Pure Bending 240
4.2 Stresses and Deformations in the Elastic Range 244
4.3 Deformations in a Transverse Cross Section 248
4.4 Members Made of Composite Materials 259
4.5 Stress Concentrations 263
*4.6 Plastic Deformations 273
4.7 Eccentric Axial Loading in a Plane of Symmetry 291
4.8 Unsymmetric Bending Analysis 303
4.9 General Case of Eccentric Axial Loading Analysis 308
*4.10 Curved Members 320
Review and Summary 335

5 Analysis and Design of Beams for Bending 347

5.1 Shear and Bending-Moment Diagrams 350
5.2 Relationships Between Load, Shear, and Bending Moment 362
5.3 Design of Prismatic Beams for Bending 373
*5.4 Singularity Functions Used to Determine Shear and Bending Moment 385
*5.5 Nonprismatic Beams 398
Review and Summary 408

6 Shearing Stresses in Beams and Thin-Walled Members 417

6.1 Horizontal Shearing Stress in Beams 420
*6.2 Distribution of Stresses in a Narrow Rectangular Beam 426
6.3 Longitudinal Shear on a Beam Element of Arbitrary Shape 437
6.4 Shearing Stresses in Thin-Walled Members 439
*6.5 Plastic Deformations 441
*6.6 Unsymmetric Loading of Thin-Walled Members and Shear Center 454
Review and Summary 467

7 Transformations of Stress and Strain 477

7.1 Transformation of Plane Stress 480
7.2 Mohr's Circle for Plane Stress 492
7.3 General State of Stress 503
7.4 Three-Dimensional Analysis of Stress 504
*7.5 Theories of Failure 507
7.6 Stresses in Thin-Walled Pressure Vessels 520
*7.7 Transformation of Plane Strain 529
*7.8 Three-Dimensional Analysis of Strain 534
*7.9 Measurements of Strain; Strain Rosette 538
Review and Summary 546

8 Principal Stresses Under a Given Loading 557

8.1 Principal Stresses in a Beam 559
8.2 Design of Transmission Shafts 562
8.3 Stresses Under Combined Loads 575
Review and Summary 591

9 Deflection of Beams 599

9.1 Deformation Under Transverse Loading 602
9.2 Statically Indeterminate Beams 611
***9.3** Singularity Functions to Determine Slope and Deflection 623
9.4 Method of Superposition 635
***9.5** Moment-Area Theorems 649
***9.6** Moment-Area Theorems Applied to Beams with Unsymmetric Loadings 664
Review and Summary 679

10 Columns 691

10.1 Stability of Structures 692
***10.2** Eccentric Loading and the Secant Formula 709
10.3 Centric Load Design 722
10.4 Eccentric Load Design 739
Review and Summary 750

11 Energy Methods 759

11.1 Strain Energy 760
11.2 Elastic Strain Energy 763
11.3 Strain Energy for a General State of Stress 770
11.4 Impact Loads 784
11.5 Single Loads 788
***11.6** Work and Energy Under Multiple Loads 802
***11.7** Castigliano's Theorem 804
***11.8** Deflections by Castigliano's Theorem 806
***11.9** Statically Indeterminate Structures 810
Review and Summary 823

Appendices A1

A Principal Units Used in Mechanics A2
B Centroids and Moments of Areas A4
C Centroids and Moments of Inertia of Common Geometric Shapes A15
D Typical Properties of Selected Materials Used in Engineering A17
E Properties of Rolled-Steel Shapes A21
F Beam Deflections and Slopes A33
G Fundamentals of Engineering Examination A34

Answers to Problems AN1
Index I1

Preface

Objectives

The main objective of a basic mechanics course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This text is designed for the first course in mechanics of materials—or strength of materials—offered to engineering students in the sophomore or junior year. The authors hope that it will help instructors achieve this goal in that particular course in the same way that their other texts may have helped them in statics and dynamics.

General Approach

In this text the study of the mechanics of materials is based on the understanding of a few basic concepts and on the use of simplified models. This approach makes it possible to develop all the necessary formulas in a rational and logical manner, and to indicate clearly the conditions under which they can be safely applied to the analysis and design of actual engineering structures and machine components.

This title is supported by SmartBook, a feature of the LearnSmart adaptive learning system that assesses student understanding of course content through a series of adaptive questions. This platform has provided feedback from thousands of students, identifying those specific portions of the text that have resulted in the greatest conceptual difficulty and comprehension among the students. For this new edition, the entire text was reviewed and revised based on this LearnSmart student data.

Additionally, over 25% of the assigned problems from the previous edition have been replaced or revised. Photographic content has also been modified to provide a more suitable conceptual context to the important principles discussed.

NEW!

Lab Videos. Student understanding of mechanics of materials is greatly enhanced through experimental study that complements the classroom experience. Due to resource and curricular constraints, however, very few engineering programs are able to provide such a laboratory component as part of a mechanics of materials course. To help address this need, this edition includes videos that show key mechanics of materials experiments being conducted. For each experiment, data has been generated so that students can analyze results and write reports. These videos are incorporated into the Connect digital platform.

Free-body Diagrams Are Used Extensively. Throughout the text free-body diagrams are used to determine external or internal forces. The use of "picture equations" will also help the students understand the superposition of loadings and the resulting stresses and deformations.

The SMART Problem-Solving Methodology Is Employed. Continuing in this edition, students are presented with the SMART approach for solving engineering problems, whose acronym reflects the solution steps of ***S***trategy,

Modeling, ***A***nalysis, and ***R***eflect & ***T***hink. This methodology is used in all Sample Problems, and it is intended that students will apply this approach in the solution of all assigned problems.

Design Concepts Are Discussed Throughout the Text When Appropriate. A discussion of the application of the factor of safety to design can be found in Chapter 1, where the concepts of both *Allowable Stress Design* and *Load and Resistance Factor Design* are presented.

A Careful Balance Between SI and U.S. Customary Units Is Consistently Maintained. Because it is essential that students be able to handle effectively both SI metric units and U.S. customary units, half the Concept Applications, Sample Problems, and problems to be assigned have been stated in SI units and half in U.S. customary units. Since a large number of problems are available, instructors can assign problems using each system of units in whatever proportion they find desirable for their class.

Optional Sections Offer Advanced or Specialty Topics. Topics such as residual stresses, torsion of noncircular and thin-walled members, bending of curved beams, shearing stresses in nonsymmetrical members, and failure criteria have been included in optional sections for use in courses of varying emphases. To preserve the integrity of the subject, these topics are presented in the proper sequence, wherever they logically belong. Thus, even when not covered in the course, these sections are highly visible and can be easily referred to by the students if needed in a later course or in engineering practice. For convenience all optional sections have been indicated by asterisks.

Chapter Organization

It is expected that students using this text will have completed a course in statics. However, Chapter 1 is designed to provide them with an opportunity to review the concepts learned in that course, while shear and bending-moment diagrams are covered in detail in Sections 5.1 and 5.2. The properties of moments and centroids of areas are described in Appendix B; this material can be used to reinforce the discussion of the determination of normal and shearing stresses in beams (Chapters 4, 5, and 6).

The first four chapters of the text are devoted to the analysis of the stresses and of the corresponding deformations in various structural members, considering successively axial loading, torsion, and pure bending. Each analysis is based on a few basic concepts: namely, the conditions of equilibrium of the forces exerted on the member, the relations existing between stress and strain in the material, and the conditions imposed by the supports and loading of the member. The study of each type of loading is complemented by a large number of Concept Applications, Sample Problems, and problems to be assigned, all designed to strengthen the students' understanding of the subject.

The concept of stress at a point is introduced in Chapter 1, where it is shown that an axial load can produce shearing stresses as well as normal stresses, depending upon the section considered. The fact that stresses depend upon the orientation of the surface on which they are computed is emphasized again in Chapters 3 and 4 in the cases of torsion and pure bending. However, the discussion of computational techniques—such as Mohr's circle—used for the transformation of stress at a point is delayed until Chapter 7, after students have had the opportunity to solve problems involving a combination of the basic loadings and have discovered for themselves the need for such techniques.

The discussion in Chapter 2 of the relation between stress and strain in various materials includes fiber-reinforced composite materials. Also, the study of beams under transverse loads is covered in two separate chapters. Chapter 5 is devoted to the determination of the normal stresses in a beam and to the design of beams based on the allowable normal stress in the material used (Section 5.3). The chapter begins with a discussion of the shear and bending-moment diagrams (Sections 5.1 and 5.2) and includes an optional section on the use of singularity functions for the determination of the shear and bending moment in a beam (Section 5.4). The chapter ends with an optional section on nonprismatic beams (Section 5.5).

Chapter 6 is devoted to the determination of shearing stresses in beams and thin-walled members under transverse loadings. The formula for the shear flow, $q = VQ/I$, is derived in the traditional way. More advanced aspects of the design of beams, such as the determination of the principal stresses at the junction of the flange and web of a W-beam, are considered in Chapter 8, an optional chapter that may be covered after the transformations of stresses have been discussed in Chapter 7. The design of transmission shafts is in that chapter for the same reason, as well as the determination of stresses under combined loadings that can now include the determination of the principal stresses, principal planes, and maximum shearing stress at a given point.

Statically indeterminate problems are first discussed in Chapter 2 and considered throughout the text for the various loading conditions encountered. Thus students are presented at an early stage with a method of solution that combines the analysis of deformations with the conventional analysis of forces used in statics. In this way, they will have become thoroughly familiar with this fundamental method by the end of the course. In addition, this approach helps the students realize that stresses themselves are statically indeterminate and can be computed only by considering the corresponding distribution of strains.

The concept of plastic deformation is introduced in Chapter 2, where it is applied to the analysis of members under axial loading. Problems involving the plastic deformation of circular shafts and of prismatic beams are also considered in optional sections of Chapters 3, 4, and 6. While some of this material can be omitted at the choice of the instructor, its inclusion in the body of the text will help students realize the limitations of the assumption of a linear stress-strain relation and serve to caution them against the inappropriate use of the elastic torsion and flexure formulas.

The determination of the deflection of beams is discussed in Chapter 9. The first part of the chapter is devoted to the integration method and to the method of superposition, with an optional section (Section 9.3) based on the use of singularity functions. (This section should be used only if Section 5.4 was covered earlier.) The second part of Chapter 9 is optional. It presents the moment-area method in two lessons.

Chapter 10, which is devoted to columns, contains material on the design of steel, aluminum, and wood columns. Chapter 11 covers energy methods, including Castigliano's theorem.

Supplemental Resources for Instructors

Included on the website are lecture PowerPoints and an image library. On the site you'll also find the Instructor's Solutions Manual (password-protected and available to instructors only) that accompanies the eighth edition. The manual continues the tradition of exceptional accuracy and normally keeps solutions contained to a single page for easier reference. The manual includes an in-depth review of the material in each chapter and houses tables designed to assist instructors in creating a schedule of assignments for their courses. The various topics covered in the text are listed in Table I, and a suggested number of periods to be spent on each topic is indicated. Table II provides a brief description of all groups of problems and a classification of the problems in each group according to the units used. A Course Organization Guide providing sample assignment schedules is also found on the website.

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We also gratefully acknowledge the help, comments, and suggestions offered by the many reviewers and users of previous editions of *Mechanics of Materials*.

John T. DeWolf
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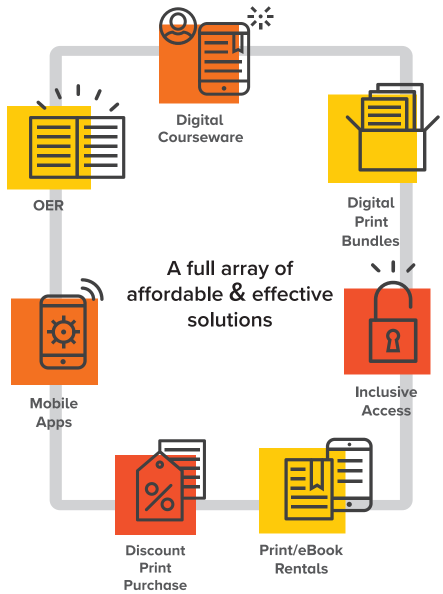

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Guided Tour

Chapter Introduction. Each chapter begins with an introductory section that sets up the purpose and goals of the chapter, describing in simple terms the material that will be covered and its application to the solution of engineering problems. Chapter Objectives provide students with a preview of chapter topics.

Chapter Lessons. The body of the text is divided into units, each consisting of one or several theory sections, Concept Applications, one or several Sample Problems, and a large number of homework problems. The Companion Website contains a Course Organization Guide with suggestions on each chapter lesson.

Concept Application 1.1

Considering the structure of Fig. 1.1, assume that rod BC is made of a steel with a maximum allowable stress $\sigma_{all} = 165$ MPa. Can rod BC safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was 50 kN. Recalling that the diameter of the rod is 20 mm, use Eq. (1.5) to determine the stress created in the rod by the given loading.

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$

$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since σ is smaller than σ_{all} of the allowable stress in the steel used, rod BC can safely support the load.

Concept Applications. Concept Applications are used extensively within individual theory sections to focus on specific topics, and they are designed to illustrate specific material being presented and facilitate its understanding.

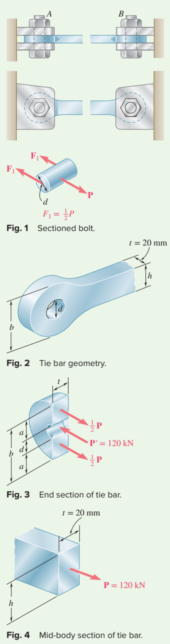


Fig. 1 Sectioned bolt.

Fig. 2 Tie bar geometry.

Fig. 3 End section of tie bar.

Fig. 4 Mid-body section of tie bar.

Sample Problem 1.2

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are $\sigma = 175$ MPa, $\tau = 100$ MPa, and $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (a) the diameter d of the bolt, (b) the dimension b at each end of the bar, and (c) the dimension h of the bar.

STRATEGY: Use free-body diagrams to determine the forces needed to obtain the stresses in terms of the design tension force. Setting these stresses equal to the allowable stresses provides for the determination of the required dimensions.

MODELING and ANALYSIS:

a. Diameter of the Bolt. Since the bolt is in double shear (Fig. 1), $F_1 = \frac{1}{2}P = 60$ kN.

$$\tau = \frac{F_1}{A} = \frac{60 \text{ kN}}{\frac{1}{4}\pi d^2} \qquad 100 \text{ MPa} = \frac{60 \text{ kN}}{\frac{1}{4}\pi d^2} \qquad d = 27.6 \text{ mm}$$

Use $d = 28$ mm ◀

At this point, check the bearing stress between the 20-mm-thick plate (Fig. 2) and the 28-mm-diameter bolt.

$$\sigma_b = \frac{P}{td} = \frac{120 \text{ kN}}{(0.020 \text{ m})(0.028 \text{ m})} = 214 \text{ MPa} < 350 \text{ MPa} \qquad \text{OK}$$

b. Dimension b at Each End of the Bar. We consider one of the end portions of the bar in Fig. 3. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, write

$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175 \text{ MPa} = \frac{60 \text{ kN}}{(0.02 \text{ m})a} \qquad a = 17.14 \text{ mm}$$

$$b = d + 2a = 28 \text{ mm} + 2(17.14 \text{ mm}) \qquad b = 62.3 \text{ mm} ◀$$

c. Dimension h of the Bar. We consider a section in the central portion of the bar (Fig. 4). Recalling that the thickness of the steel plate is $t = 20$ mm, we have

$$\sigma = \frac{P}{th} \qquad 175 \text{ MPa} = \frac{120 \text{ kN}}{(0.020 \text{ m})h} \qquad h = 34.3 \text{ mm}$$

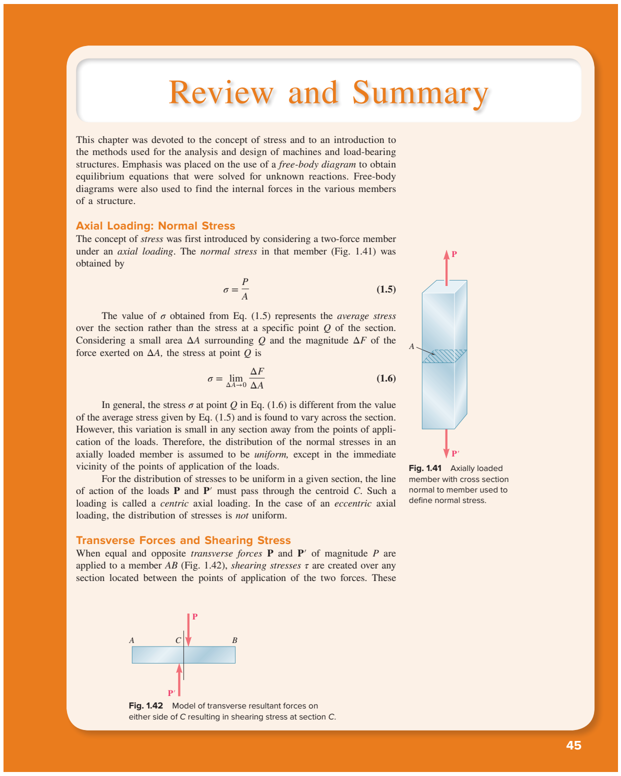
Use $h = 35$ mm ◀

REFLECT and THINK: We sized d based on bolt shear, and then checked bearing on the tie bar. Had the maximum allowable bearing stress been exceeded, we would have had to recalculate d based on the bearing criterion.

Sample Problems. The Sample Problems are intended to show more comprehensive applications of the theory to the solution of engineering problems, and they employ the SMART problem-solving methodology that students are encouraged to use in the solution of their assigned problems. Since the sample problems have been set up in much the same form that students will use in solving the assigned problems, they serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. In addition, in-problem references and captions have been added to the sample problem figures for contextual linkage to the step-by-step solution.

Homework Problem Sets. Over 25% of the nearly 1500 homework problems are new or updated. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and to help students understand the principles used in mechanics of materials. The problems are grouped according to the portions of material they illustrate and are arranged in order of increasing difficulty. Answers to a majority of the problems are given at the end of the book. Problems for which the answers are given are set in blue type in the text, while problems for which no answer is given are set in red italics.

Chapter Review and Summary. Each chapter ends with a review and summary of the material covered in that chapter. Subtitles are used to help students organize their review work, and cross-references have been included to help them find the portions of material requiring their special attention.



Review and Summary

This chapter was devoted to the concept of stress and to an introduction to the methods used for the analysis and design of machines and load-bearing structures. Emphasis was placed on the use of a *free-body diagram* to obtain equilibrium equations that were solved for unknown reactions. Free-body diagrams were also used to find the internal forces in the various members of a structure.

Axial Loading: Normal Stress

The concept of *stress* was first introduced by considering a two-force member under an *axial loading*. The *normal stress* in that member (Fig. 1.41) was obtained by

$$\sigma = \frac{P}{A} \quad (1.5)$$

The value of σ obtained from Eq. (1.5) represents the *average stress* over the section rather than the stress at a specific point Q of the section. Considering a small area ΔA surrounding Q and the magnitude ΔF of the force exerted on ΔA, the stress at point Q is

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad (1.6)$$

In general, the stress σ at point Q in Eq. (1.6) is different from the value of the average stress given by Eq. (1.5) and is found to vary across the section. However, this variation is small in any section away from the points of application of the loads. Therefore, the distribution of the normal stresses in an axially loaded member is assumed to be *uniform*, except in the immediate vicinity of the points of application of the loads.

For the distribution of stresses to be uniform in a given section, the line of action of the loads **P** and **P′** must pass through the centroid C. Such a loading is called a *centric* axial loading. In the case of an *eccentric* axial loading, the distribution of stresses is *not* uniform.

Fig. 1.41 Axially loaded member with cross section normal to member used to define normal stress.

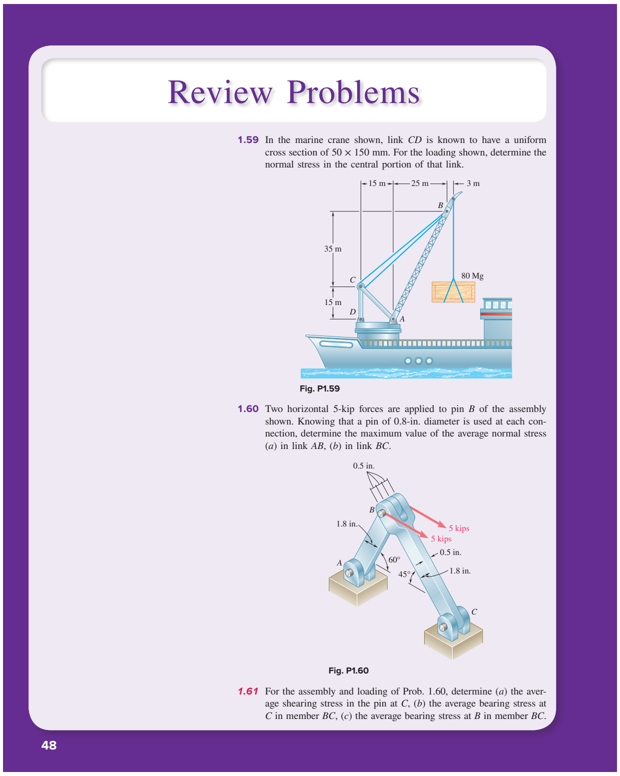
Transverse Forces and Shearing Stress

When equal and opposite *transverse forces* **P** and **P′** of magnitude P are applied to a member AB (Fig. 1.42), *shearing stresses* τ are created over any section located between the points of application of the two forces. These

Fig. 1.42 Model of transverse resultant forces on either side of C resulting in shearing stress at section C.

45

Review Problems. A set of review problems is included at the end of each chapter. These problems provide students further opportunity to apply the most important concepts introduced in the chapter.



Review Problems

1.59 In the marine crane shown, link CD is known to have a uniform cross section of 50 × 150 mm. For the loading shown, determine the normal stress in the central portion of that link.

Fig. P1.59

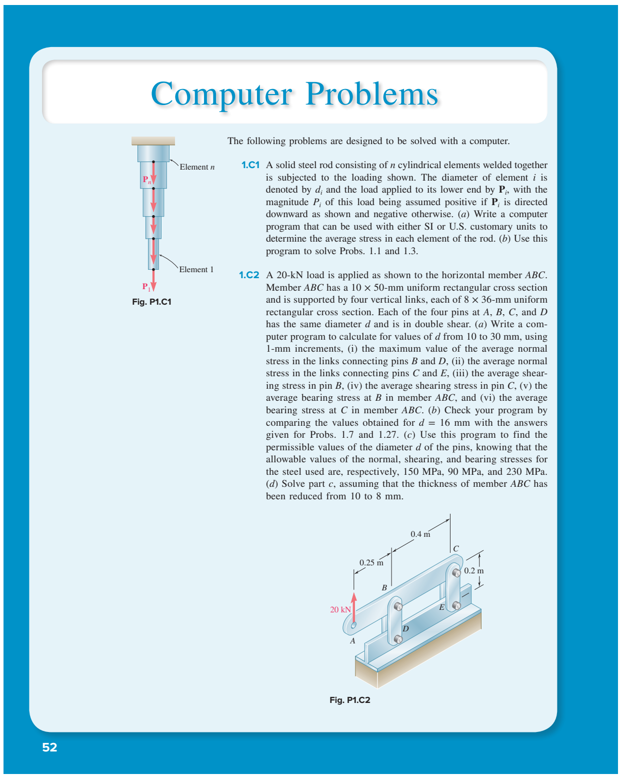
1.60 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (*a*) in link AB, (*b*) in link BC.

Fig. P1.60

1.61 For the assembly and loading of Prob. 1.60, determine (*a*) the average shearing stress in the pin at C, (*b*) the average bearing stress at C in member BC, (*c*) the average bearing stress at B in member BC.

48

Computer Problems. Computers make it possible for engineering students to solve a great number of challenging problems. A group of six or more problems designed to be solved with a computer can be found at the end of each chapter. These problems can be solved using any computer language that provides a basis for analytical calculations. Developing the algorithm required to solve a given problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply the skills acquired in their computer programming course to the solution of a meaningful engineering problem.



Computer Problems

The following problems are designed to be solved with a computer.

1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program that can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.1 and 1.3.

Fig. P1.C1

1.C2 A 20-kN load is applied as shown to the horizontal member ABC. Member ABC has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at A, B, C, and D has the same diameter d and is in double shear. (*a*) Write a computer program to calculate for values of d from 10 to 30 mm, using 1-mm increments, (i) the maximum value of the average normal stress in the links connecting pins B and D, (ii) the average normal stress in the links connecting pins C and E, (iii) the average shearing stress in pin B, (iv) the average shearing stress in pin C, (v) the average bearing stress at B in member ABC, and (vi) the average bearing stress at C in member ABC. (*b*) Check your program by comparing the values obtained for d = 16 mm with the answers given for Probs. 1.7 and 1.27. (*c*) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part c, assuming that the thickness of member ABC has been reduced from 10 to 8 mm.

Fig. P1.C2

52

List of Symbols

a	Constant; distance
$\mathbf{A, B, C}, \ldots$	Forces; reactions
$A, B, C, \ldots$	Points
$A, \mathcal{A}$	Area
b	Distance; width
c	Constant; distance; radius
C	Centroid
$C_1, C_2, \ldots$	Constants of integration
C_P	Column stability factor
d	Distance; diameter; depth
D	Diameter
e	Distance; eccentricity; dilatation
E	Modulus of elasticity
f	Frequency; function
$\mathbf{F}$	Force
$F.S.$	Factor of safety
G	Modulus of rigidity; shear modulus
h	Distance; height
$\mathbf{H}$	Force
H, J, K	Points
$I, I_x, \ldots$	Moment of inertia
$I_{xy}, \ldots$	Product of inertia
J	Polar moment of inertia
k	Spring constant; shape factor; bulk modulus; constant
K	Stress concentration factor; torsional spring constant
l	Length; span
L	Length; span
L_e	Effective length
m	Mass
$\mathbf{M}$	Couple
$M, M_x, \ldots$	Bending moment
M_D	Bending moment, dead load (LRFD)
M_L	Bending moment, live load (LRFD)
M_U	Bending moment, ultimate load (LRFD)
n	Number; ratio of moduli of elasticity; normal direction
p	Pressure
$\mathbf{P}$	Force; concentrated load
P_D	Dead load (LRFD)
P_L	Live load (LRFD)
P_U	Ultimate load (LRFD)
q	Shearing force per unit length; shear flow
$\mathbf{Q}$	Force
Q	First moment of area
r	Radius; radius of gyration
$\mathbf{R}$	Force; reaction
R	Radius; modulus of rupture
s	Length
S	Elastic section modulus
t	Thickness; distance; tangential deviation
$\mathbf{T}$	Torque
T	Temperature
u, v	Rectangular coordinates
u	Strain-energy density
U	Strain energy; work
$\mathbf{v}$	Velocity
$\mathbf{V}$	Shearing force
V	Volume; shear
w	Width; distance; load per unit length
$\mathbf{W}, W$	Weight, load
x, y, z	Rectangular coordinates; distance; displacements; deflections
$\bar{x}, \bar{y}, \bar{z}$	Coordinates of centroid
Z	Plastic section modulus
α, β, γ	Angles
α	Coefficient of thermal expansion; influence coefficient
γ	Shearing strain; specific weight
γ_D	Load factor, dead load (LRFD)
γ_L	Load factor, live load (LRFD)
δ	Deformation; displacement
ε	Normal strain
θ	Angle; slope
λ	Direction cosine
ν	Poisson's ratio
ρ	Radius of curvature; distance; density
σ	Normal stress
τ	Shearing stress
ϕ	Angle; angle of twist; resistance factor
ω	Angular velocity

Eighth Edition

Mechanics of Materials

1

Introduction—Concept of Stress

Stresses occur in all structures subject to loads. This chapter will examine simple states of stress in elements, such as in the two-force members, bolts, and pins used in the structure shown.

Objectives

In this chapter, we will:

- **Review statics** needed to determine forces in members of simple structures.
- **Introduce** the concept of stress.
- **Define** different stress types: axial normal stress, shearing stress, and bearing stress.
- **Discuss** an engineer's two principal tasks: the analysis and design of structures and machines.
- **Develop** a problem-solving approach.
- **Discuss** the components of stress on different planes and under different loading conditions.
- **Discuss** the many design considerations that an engineer should review before preparing a design.

Introduction

1.1 REVIEW OF THE METHODS OF STATICS

1.2 STRESSES IN THE MEMBERS OF A STRUCTURE

1.2A Axial Stress

1.2B Shearing Stress

1.2C Bearing Stress in Connections

1.2D Application to the Analysis and Design of Simple Structures

1.2E Method of Problem Solution

1.3 STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING

1.4 STRESS UNDER GENERAL LOADING CONDITIONS; COMPONENTS OF STRESS

1.5 DESIGN CONSIDERATIONS

1.5A Determination of the Ultimate Strength of a Material

1.5B Allowable Load and Allowable Stress: Factor of Safety

1.5C Factor of Safety Selection

1.5D Load and Resistance Factor Design

Introduction

The study of mechanics of materials provides future engineers with the means of analyzing and designing various machines and load-bearing structures involving the determination of *stresses* and *deformations*. This first chapter is devoted to the concept of *stress*.

Section 1.1 is a short review of the basic methods of statics and their application to determine the forces in the members of a simple structure consisting of pin-connected members. The concept of *stress* in a member of a structure and how that stress can be determined from the *force* in the member will be discussed in Sec. 1.2. You will consider the *normal stresses* in a member under axial loading, the *shearing stresses* caused by the application of equal and opposite transverse forces, and the *bearing stresses* created by bolts and pins in the members they connect.

Section 1.2 ends with a description of the method you should use in the solution of an assigned problem and a discussion of the numerical accuracy. These concepts will be applied in the analysis of the members of the simple structure considered earlier.

Again, a two-force member under axial loading is observed in Sec. 1.3 where the stresses on an *oblique* plane include both *normal* and *shearing* stresses, while Sec. 1.4 discusses that *six components* are required to describe the state of stress at a point in a body under the most general loading conditions.

Section 1.5 is devoted to the determination of the *ultimate strength* from test specimens and the use of a *factor of safety* to compute the *allowable load* for a structural component made of that material.

1.1 REVIEW OF THE METHODS OF STATICS

Consider the structure shown in Fig. 1.1, which was designed to support a 30-kN load. It consists of a boom AB with a 30 × 50-mm rectangular cross section and a rod BC with a 20-mm-diameter circular cross section. These are connected by a pin at B and are supported by pins and brackets at A and C, respectively. First draw a *free-body diagram* of the structure. This is done by detaching the structure from its supports at A and C, and then showing the reactions that these supports exert on the structure (Fig. 1.2). Note that the sketch of the structure has been simplified by omitting all unnecessary details. Many of you may have recognized at this point that AB and BC are *two-force members*. For those of you who have not, we will pursue our analysis, ignoring that fact and assuming that the directions of the reactions at A and C are unknown. Each of these reactions are represented by two components: $\mathbf{A}_x$ and $\mathbf{A}_y$ at A, and $\mathbf{C}_x$ and $\mathbf{C}_y$ at C. The three equilibrium equations are.

$$+\circlearrowleft \Sigma M_C = 0: \qquad A_x(0.6 \text{ m}) - (30 \text{ kN})(0.8 \text{ m}) = 0$$

$$A_x = +40 \text{ kN} \qquad \textbf{(1.1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad A_x + C_x = 0$$

$$C_x = -A_x \qquad C_x = -40 \text{ kN} \qquad \textbf{(1.2)}$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + C_y - 30 \text{ kN} = 0$$

$$A_y + C_y = +30 \text{ kN} \qquad \textbf{(1.3)}$$

Photo 1.1 Crane booms used to load and unload ships.

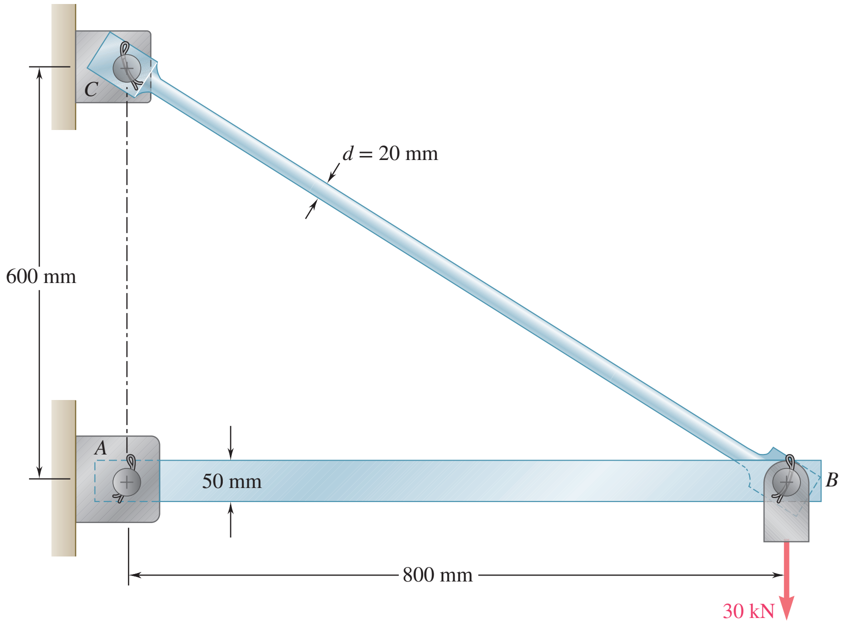


Fig. 1.1 Boom used to support a 30-kN load.

We have found two of the four unknowns, but cannot determine the other two from these equations, and no additional independent equation can be obtained from the free-body diagram of the structure. We must now dismember the structure. Considering the free-body diagram of the boom *AB* (Fig. 1.3), we write the following equilibrium equation:

$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(0.8 \text{ m}) = 0 \qquad A_y = 0 \qquad \textbf{(1.4)}$$

Substituting for A_y from Eq. (1.4) into Eq. (1.3), we obtain $C_y = +30$ kN. Expressing the results obtained for the reactions at *A* and *C* in vector form, we have

$$\mathbf{A} = 40 \text{ kN} \rightarrow \qquad \mathbf{C}_x = 40 \text{ kN} \leftarrow \qquad \mathbf{C}_y = 30 \text{ kN} \uparrow$$

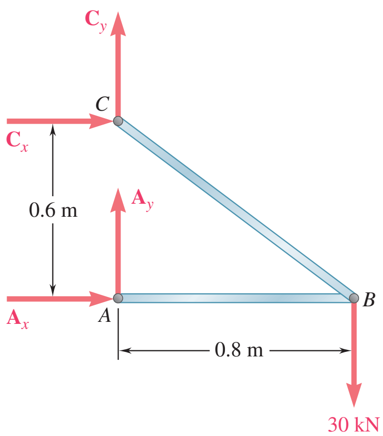


Fig. 1.2 Free-body diagram of boom showing applied load and reaction forces.

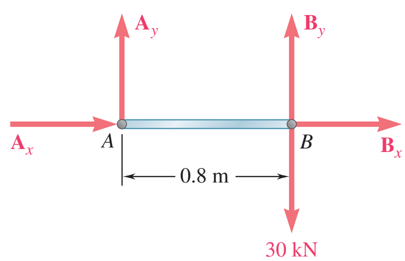


Fig. 1.3 Free-body diagram of member *AB* freed from structure.

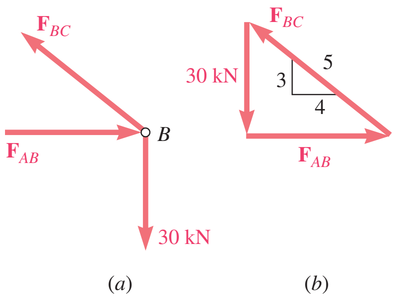


Fig. 1.4 Free-body diagram of boom's joint *B* and associated force triangle.

Note that the reaction at *A* is directed along the axis of the boom *AB* and causes compression in that member. Observe that the components C_x and C_y of the reaction at *C* are, respectively, proportional to the horizontal and vertical components of the distance from *B* to *C* and that the reaction at *C* is equal to 50 kN, is directed along the axis of the rod *BC*, and causes tension in that member.

These results could have been anticipated by recognizing that *AB* and *BC* are two-force members, i.e., members that are subjected to forces at only two points, these points being *A* and *B* for member *AB*, and *B* and *C* for member *BC*. Indeed, for a two-force member the lines of action of the resultants of the forces acting at each of the two points are equal and opposite and pass through both points. Using this property, we could have obtained a simpler solution by considering the free-body diagram of pin *B*. The forces on pin *B*, $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$, are exerted, respectively, by members *AB* and *BC* and the 30-kN load (Fig. 1.4*a*). Pin *B* is shown to be in equilibrium by drawing the corresponding force triangle (Fig. 1.4*b*).

Since force $\mathbf{F}_{BC}$ is directed along member *BC*, its slope is the same as that of *BC*, namely, 3/4. We can, therefore, write the proportion

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30 \text{ kN}}{3}$$

from which

$$F_{AB} = 40 \text{ kN} \qquad F_{BC} = 50 \text{ kN}$$

Forces $\mathbf{F}'_{AB}$ and $\mathbf{F}'_{BC}$ exerted by pin *B* on boom *AB* and rod *BC* are equal and opposite to $\mathbf{F}_{AB}$ and $\mathbf{F}_{BC}$ (Fig. 1.5).

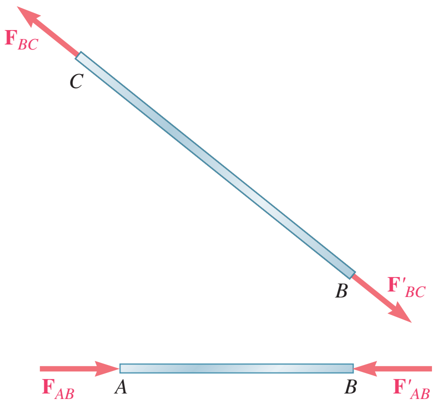


Fig. 1.5 Free-body diagrams of two-force members *AB* and *BC*.

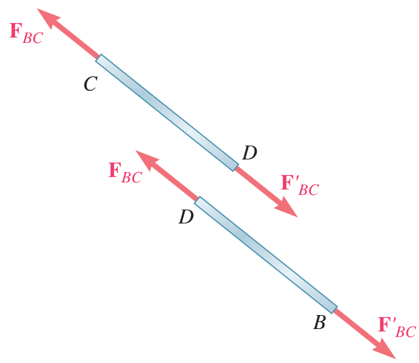


Fig. 1.6 Free-body diagrams of sections of rod *BC*.

Knowing the forces at the ends of each member, we can now determine the internal forces in these members. Passing a section at some arbitrary point *D* of rod *BC*, we obtain two portions *BD* and *CD* (Fig. 1.6). Since 50-kN forces must be applied at *D* to both portions of the rod to keep them in equilibrium, an internal force of 50 kN is produced in rod *BC* when a 30-kN load is applied at *B*. From the directions of the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ in Fig. 1.6 we see that the rod is in tension. A similar procedure enables us to determine that the internal force in boom *AB* is 40 kN and is in compression.

1.2 STRESSES IN THE MEMBERS OF A STRUCTURE

1.2A Axial Stress

In the preceding section, we found forces in individual members. This is the first and necessary step in the analysis of a structure. However it does not tell us whether the given load can be safely supported. It is necessary to look at each individual member separately to determine if the structure is safe. Rod *BC* of the example considered in the preceding section is a two-force member and, therefore, the forces $\mathbf{F}_{BC}$ and $\mathbf{F}'_{BC}$ acting on its ends *B* and *C* (Fig. 1.5) are directed along the axis of the rod. Whether rod *BC* will break or not under this loading depends upon the value found for the internal force F_{BC}, the cross-sectional area of the rod, and the material of which the rod is made. Actually, the internal force F_{BC} represents the resultant of elementary forces distributed over the entire area *A* of the cross section (Fig. 1.7). The average intensity of these distributed forces is equal to the force per unit area, F_{BC}/A, on the section. Whether or not the rod will break under the given loading depends upon the ability of the material to withstand the corresponding value F_{BC}/A of the intensity of the distributed internal forces.

Photo 1.2 This bridge truss consists of two-force members that may be in tension or in compression. ©Natalia Bratslavsky/Shutterstock

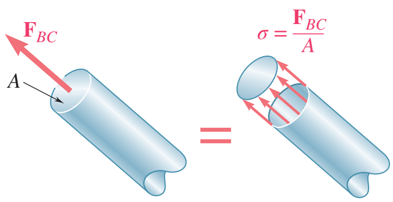


Fig. 1.7 Axial force represents the resultant of distributed elementary forces.

Using Fig. 1.8, the force per unit area is called the *stress* and is denoted by the Greek letter σ (sigma). The stress in a member of cross-sectional area *A* subjected to an axial load **P** is obtained by dividing the magnitude *P* of the load by the area *A*:

$$\sigma = \frac{P}{A} \tag{1.5}$$

A positive sign indicates a tensile stress (member in tension), and a negative sign indicates a compressive stress (member in compression).

As shown in Fig. 1.8, the section through the rod to determine the internal force in the rod and the corresponding stress is perpendicular to the axis of the rod. The corresponding stress is described as a *normal stress*. Thus, Eq. (1.5) gives the *normal stress in a member under axial loading*.

Note that in Eq. (1.5), σ represents the *average value* of the stress over the cross section, rather than the stress at a specific point of the cross section. To define the stress at a given point *Q* of the cross section, consider a small area ΔA (Fig. 1.9). Dividing the magnitude of $\Delta\mathbf{F}$ by ΔA, you obtain the average value of the stress over ΔA. Letting ΔA approach zero, the stress at point *Q* is

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \tag{1.6}$$

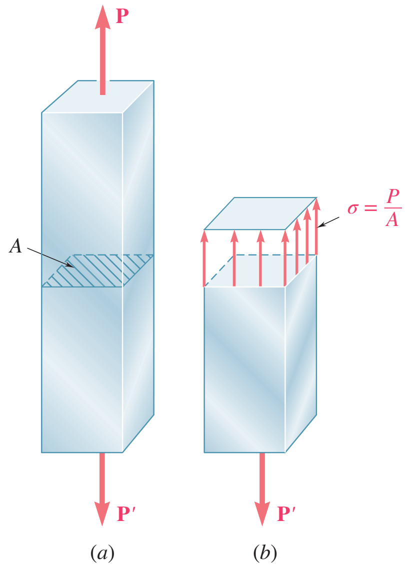


Fig. 1.8 (a) Member with an axial load. (b) Idealized uniform stress distribution at an arbitrary section.

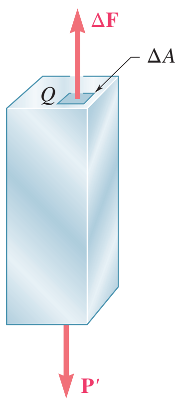


Fig. 1.9 Small area ΔA, at an arbitrary point in the cross section, carries ΔF in this axial member.

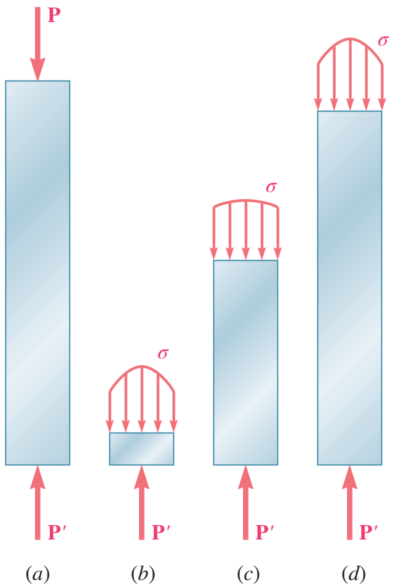


Fig. 1.10 Stress distributions at different sections along axially loaded member.

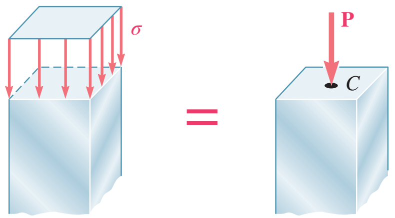


Fig. 1.11 Idealized uniform stress distribution implies the resultant force passes through the cross section's center.

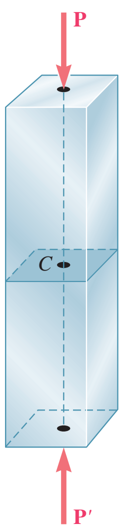


Fig. 1.12 Centric loading having resultant forces passing through the centroid of the section.

In general, the value for the stress σ at a given point Q of the section is different from that for the average stress given by Eq. (1.5), and σ is found to vary across the section. In a slender rod subjected to equal and opposite concentrated loads **P** and **P′** (Fig. 1.10*a*), this variation is small in a section away from the points of application of the concentrated loads (Fig. 1.10*c*), but it is quite noticeable in the neighborhood of these points (Fig. 1.10*b* and *d*).

It follows from Eq. (1.6) that the magnitude of the resultant of the distributed internal forces is

$$\int dF = \int_A \sigma \, dA$$

But the conditions of equilibrium of each of the portions of rod shown in Fig. 1.10 require that this magnitude be equal to the magnitude P of the concentrated loads. Therefore,

$$P = \int dF = \int_A \sigma \, dA \tag{1.7}$$

which means that the volume under each of the stress surfaces in Fig. 1.10 must be equal to the magnitude P of the loads. However, this is the only information derived from statics regarding the distribution of normal stresses in the various sections of the rod. The actual distribution of stresses in any given section is *statically indeterminate*. To learn more about this distribution, it is necessary to consider the deformations resulting from the particular mode of application of the loads at the ends of the rod. This will be discussed further in Chap. 2.

In practice, it is assumed that the distribution of normal stresses in an axially loaded member is uniform, except in the immediate vicinity of the points of application of the loads. The value σ of the stress is then equal to σ_{ave} and can be obtained from Eq. (1.5). However, realize that when we assume a uniform distribution of stresses in the section, it follows from elementary statics[†] that the resultant **P** of the internal forces must be applied at the centroid C of the section (Fig. 1.11). This means that *a uniform distribution of stress is possible only if the line of action of the concentrated loads* **P** *and* **P′** *passes through the centroid of the section considered* (Fig. 1.12). This type of loading is called *centric loading* and will take place in all straight two-force members found in trusses and pin-connected structures, such as the one considered in Fig. 1.1. However, if a two-force member is loaded axially, but *eccentrically,* as shown in Fig. 1.13*a*, the conditions of equilibrium of the portion of member in Fig. 1.13*b* show that the internal forces in a given section must be equivalent to a force **P** applied at the centroid of the section and a couple **M** of moment $M = Pd$. This distribution of forces—the corresponding distribution of stresses—*cannot be uniform*. Nor can the distribution of stresses be symmetric. This point will be discussed in detail in Chap. 4.

[†]See Ferdinand P. Beer and E. Russell Johnston, Jr., *Mechanics for Engineers,* 5th ed., McGraw-Hill, New York, 2008, or *Vector Mechanics for Engineers,* 12th ed., McGraw-Hill, New York, 2019, Sec. 5.1.

The units associated with stresses are as follows: When SI metric units are used, P is expressed in newtons (N) and A in square meters (m^2), so the stress σ will be expressed in N/m^2. This unit is called a *pascal* (Pa). However, the pascal is an exceedingly small quantity and often multiples of this unit must be used: the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa):

$$1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$$
$$1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2$$
$$1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$$

When U.S. customary units are used, force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A is given in square inches (in^2). The stress σ then is expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).†

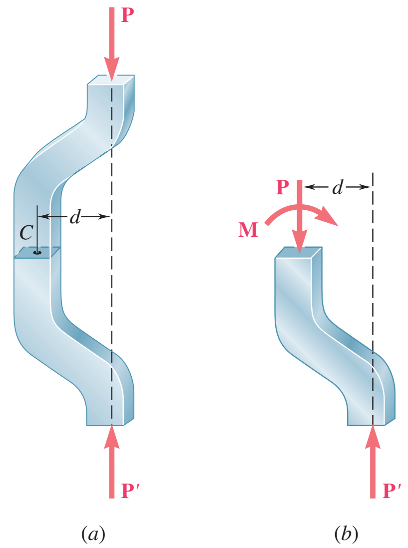


Fig. 1.13 An example of eccentric loading.

Concept Application 1.1

Considering the structure of Fig. 1.1, assume that rod BC is made of a steel with a maximum allowable stress $\sigma_{\text{all}} = 165$ MPa. Can rod BC safely support the load to which it will be subjected? The magnitude of the force F_{BC} in the rod was 50 kN. Recalling that the diameter of the rod is 20 mm, use Eq. (1.5) to determine the stress created in the rod by the given loading.

$$P = F_{BC} = +50 \text{ kN} = +50 \times 10^3 \text{ N}$$

$$A = \pi r^2 = \pi\left(\frac{20 \text{ mm}}{2}\right)^2 = \pi(10 \times 10^{-3} \text{ m})^2 = 314 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{+50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = +159 \times 10^6 \text{ Pa} = +159 \text{ MPa}$$

Since σ is smaller than σ_{all} of the allowable stress in the steel used, rod BC can safely support the load.

To be complete, our analysis of the given structure should also include the compressive stress in boom AB, as well as the stresses produced in the pins and their bearings. This will be discussed later in this chapter. You should also determine whether the deformations produced by the given loading are acceptable. The study of deformations under axial loads will be the subject of Chap. 2. For members in compression, the *stability* of the member (i.e., its ability to support a given load without experiencing a sudden change in configuration) will be discussed in Chap. 10.

†The principal SI and U.S. customary units used in mechanics are listed in Appendix A. Using the third table, 1 psi is approximately equal to 7 kPa, and 1 ksi is approximately equal to 7 MPa.