

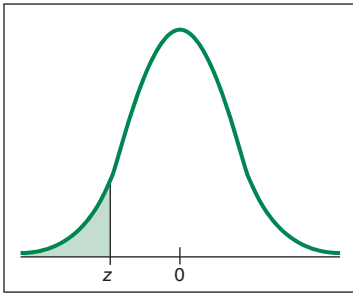
# Understandable Statistics

Concepts and  
Methods



11th Edition

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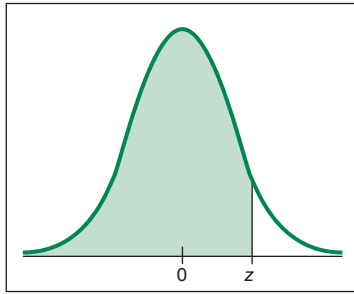


The table entry for  $z$  is the area to the left of  $z$ .

## Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of $z$										
$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

For values of  $z$  less than  $-3.49$ , use  $0.000$  to approximate the area.



The table entry for  $z$  is the area to the left of  $z$ .

### Areas of a Standard Normal Distribution *continued*

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

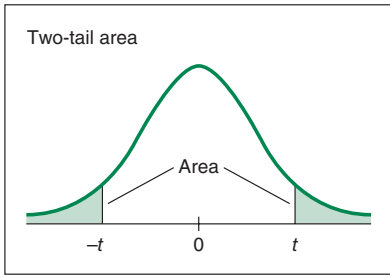
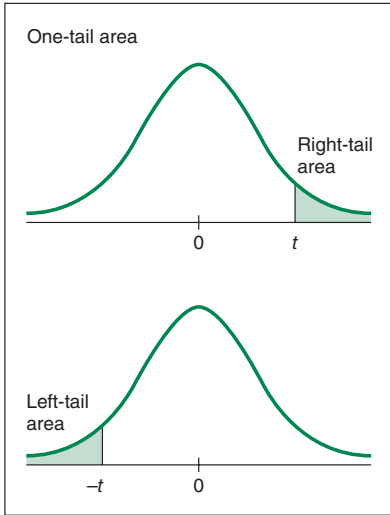
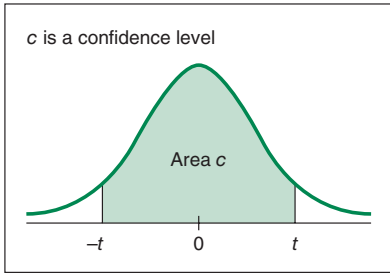
For  $z$  values greater than 3.49, use 1.000 to approximate the area.

### Areas of a Standard Normal Distribution *continued*

(b) Confidence Interval Critical Values $z_c$	
Level of Confidence $c$	Critical Value $z_c$
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

### Areas of a Standard Normal Distribution *continued*

(c) Hypothesis Testing, Critical Values $z_0$		
Level of Significance	$\alpha = 0.05$	$\alpha = 0.01$
Critical value $z_0$ for a left-tailed test	-1.645	-2.33
Critical value $z_0$ for a right-tailed test	1.645	2.33
Critical values $\pm z_0$ for a two-tailed test	$\pm 1.96$	$\pm 2.58$



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### Critical Values for Student's *t* Distribution

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005	
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010	
<i>d.f.</i>	<i>c</i>	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1		1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2		0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3		0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4		0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5		0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6		0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7		0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8		0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9		0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10		0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11		0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12		0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13		0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14		0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15		0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16		0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17		0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18		0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19		0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20		0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21		0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22		0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23		0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24		0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25		0.684	1.178	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26		0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27		0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28		0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29		0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30		0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35		0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40		0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45		0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50		0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60		0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70		0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80		0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100		0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500		0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000		0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
$\infty$		0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom *d.f.* not in the table, use the closest *d.f.* that is *smaller*.



ELEVENTH EDITION

# UNDERSTANDABLE STATISTICS

Concepts and Methods

**Charles Henry Brase**  
*Regis University*

**Corrinne Pellillo Brase**  
*Arapahoe Community College*



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*This book is dedicated to the memory of  
a great teacher, mathematician, and friend*

**Burton W. Jones**  
**Professor Emeritus, University of Colorado**



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# CRITICAL THINKING

Students need to develop critical thinking skills in order to understand and evaluate the limitations of statistical methods. *Understandable Statistics: Concepts and Methods* makes students aware of method appropriateness, assumptions, biases, and justifiable conclusions.

## CRITICAL THINKING

### UNUSUAL VALUES

Chebyshev's theorem tells us that no matter what the data distribution looks like, at least 75% of the data will fall within 2 standard deviations of the mean. As we will see in Chapter 6, when the distribution is mound-shaped and symmetrical, about 95% of the data are within 2 standard deviations of the mean. Data values beyond 2 standard deviations from the mean are less common than those closer to the mean.

In fact, one indicator that a data value might be an outlier is that it is more than 2.5 standard deviations from the mean (Source: *Statistics*, by G. Upton and I. Cook, Oxford University Press).

### UNUSUAL VALUES

For a binomial distribution, it is unusual for the number of successes  $r$  to be higher than  $\mu + 2.5\sigma$  or lower than  $\mu - 2.5\sigma$ .

We can use this indicator to determine whether a specified number of successes out of  $n$  trials in a binomial experiment are unusual.

For instance, consider a binomial experiment with 20 trials for which probability of success on a single trial is  $p = 0.70$ . The expected number of successes is  $\mu = 14$ , with a standard deviation of  $\sigma \approx 2$ . A number of successes above 19 or below 9 would be considered unusual. However, such numbers of successes are possible.

## ◀ Critical Thinking

Critical thinking is an important skill for students to develop in order to avoid reaching misleading conclusions. The Critical Thinking feature provides additional clarification on specific concepts as a safeguard against incorrect evaluation of information.

## Interpretation ▶

Increasingly, calculators and computers are used to generate the numeric results of a statistical process. However, the student still needs to correctly interpret those results in the context of a particular application. The Interpretation feature calls attention to this important step. Interpretation is stressed in examples, in guided exercises, and in the problem sets.

**SOLUTION:** Since we want to know the number of standard deviations from the mean, we want to convert 6.9 to standard  $z$  units.

$$z = \frac{x - \mu}{\sigma} = \frac{6.9 - 8}{0.5} = -2.20$$

**Interpretation** The amount of cheese on the selected pizza is only 2.20 standard deviations below the mean. The fact that  $z$  is negative indicates that the amount of cheese is 2.20 standard deviations *below* the mean. The parlor will not lose its franchise based on this sample.

- Interpretation** A campus performance series features plays, music groups, dance troops, and stand-up comedy. The committee responsible for selecting the performance groups include three students chosen at random from a pool of volunteers. This year the 30 volunteers came from a variety of majors. However, the three students for the committee were all music majors. Does this fact indicate there was bias in the selection process and that the selection process was not random? Explain.
- Critical Thinking** Greg took a random sample of size 100 from the population of current season ticket holders to State College men's basketball games. Then he took a random sample of size 100 from the population of current season ticket holders to State College women's basketball games.
  - What sampling technique (stratified, systematic, cluster, multistage, convenience, random) did Greg use to sample from the population of current season ticket holders to all State College basketball games played by either men or women?
  - Is it appropriate to pool the samples and claim to have a random sample of size 200 from the population of current season ticket holders to all State College home basketball games played by either men or women? Explain.

## ◀ NEW! Critical Thinking and Interpretation Exercises

In every section and chapter problem set, Critical Thinking problems provide students with the opportunity to test their understanding of the application of statistical methods and their interpretation of their results. Interpretation problems ask students to apply statistical results to the particular application.

# STATISTICAL LITERACY

No language, including statistics, can be spoken without learning the vocabulary. *Understandable Statistics: Concepts and Methods* introduces statistical terms with deliberate care.

## WHAT DOES THE LEVEL OF MEASUREMENT TELL US?

The level of measurement tells us which arithmetic processes are appropriate for the data. This is important because different statistical processes require various kinds of arithmetic. In some instances all we need to do is count the number of data that meet specified criteria. In such cases nominal (and higher) data levels are all appropriate. In other cases we need to order the data, so nominal data would not be suitable. Many other statistical processes require division, so data need to be at the ratio level. Just keep the nature of the data in mind before beginning statistical computations.

## ◀ NEW! What Does (concept, method, statistical result) Tell Us?

This feature gives a brief summary of the information we obtain from the named concept, method, or statistical result.

## NEW! Important Features of a (concept, method, or result) ▶

In statistics we use many different types of graphs, samples, data, and analytical methods. The features of each such tool help us select the most appropriate ones to use and help us interpret the information we receive from applications of the tools.

## IMPORTANT FEATURES OF A SIMPLE RANDOM SAMPLE

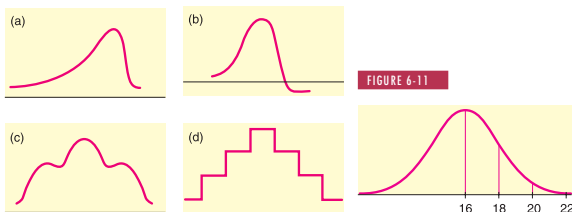
For a simple random sample

- Every sample of specified size  $n$  from the population has an equal chance of being selected.
- No researcher bias occurs in the items selected for the sample.
- A random sample may not always reflect the diversity of the population. For instance, from a population of 10 cats and 10 dogs, a random sample of size 6 could consist of all cats.

## SECTION 6.1 PROBLEMS

1. **Statistical Literacy** Which, if any, of the curves in Figure 6-10 look(s) like a normal curve? If a curve is not a normal curve, tell why.
2. **Statistical Literacy** Look at the normal curve in Figure 6-11, and find  $\mu$ ,  $\mu + \sigma$ , and  $\sigma$ .

FIGURE 6-10



## ◀ Statistical Literacy Problems

In every section and chapter problem set, Statistical Literacy problems test student understanding of terminology, statistical methods, and the appropriate conditions for use of the different processes.

## Definition Boxes ▶

Whenever important terms are introduced in text, tan definition boxes appear within the discussions. These boxes make it easy to reference or review terms as they are used further.

## BOX-AND-WHISKER PLOTS

The quartiles together with the low and high data values give us a very useful *five-number summary* of the data and their spread.

## FIVE-NUMBER SUMMARY

Lowest value,  $Q_1$ , median,  $Q_3$ , highest value

We will use these five numbers to create a graphic sketch of the data called a *box-and-whisker plot*. Box-and-whisker plots provide another useful technique from exploratory data analysis (EDA) for describing data.

# STATISTICAL LITERACY

IMPORTANT WORDS & SYMBOLS	Section 4.1	Section 4.2	Section 4.3
	Multiplication rules of probability (for independent and dependent events) 156 More than two independent events 161 Probability of $A$ or $B$ 161 Event $A$ and $B$ 161 Event $A$ or $B$ 161 Mutually exclusive events 163 Addition rules (for mutually exclusive and general events) 163 More than two mutually exclusive events 165 Basic probability rules 168	Independent events 156 Dependent events 156 Probability of $A$ and $B$ 156 Event $A B$ Conditional probability 156 $P(A B)$ 156	Multiplication rule of counting 177 Tree diagram 178 Factorial notation 181 Permutations rule 181 Combinations rule 183
	Probability of an event $A$ , $P(A)$ 144 Intuition 144 Relative frequency 144 Equally likely outcomes 144 Law of large numbers 146 Statistical experiment 146 Event 146 Simple event 146 Sample space 146 Complement of event $A^c$ 148		

## ◀ Important Words & Symbols

The Important Words & Symbols within the Chapter Review feature at the end of each chapter summarizes the terms introduced in the Definition Boxes for student review at a glance. Page numbers for first occurrence of term are given for easy reference.

## Linking Concepts: Writing Projects ▶

Much of statistical literacy is the ability to communicate concepts effectively. The Linking Concepts: Writing Projects feature at the end of each chapter tests both statistical literacy and critical thinking by asking the student to express their understanding in words.

### LINKING CONCEPTS: WRITING PROJECTS

Discuss each of the following topics in class or review the topics on your own. Then write a brief but complete essay in which you summarize the main points. Please include formulas and graphs as appropriate.

1. What does it mean to say that we are going to use a sample to draw an inference about a population? Why is a random sample so important for this process? If we wanted a random sample of students in the cafeteria, why couldn't we just choose the students who order Diet Pepsi with their lunch? Comment on the statement, "A random sample is like a miniature population, whereas samples that are not random are likely to be biased." Why would the students who order Diet Pepsi with lunch not be a random sample of students in the cafeteria?
2. In your own words, explain the differences among the following sampling techniques: simple random sample, stratified sample, systematic sample, cluster sample, multistage sample, and convenience sample. Describe situations in which each type might be useful.

5. **Basic Computation: Central Limit Theorem** Suppose  $x$  has a distribution with a mean of 8 and a standard deviation of 16. Random samples of size  $n = 64$  are drawn.
  - (a) Describe the  $\bar{x}$  distribution and compute the mean and standard deviation of the distribution.
  - (b) Find the  $z$  value corresponding to  $\bar{x} = 9$ .
  - (c) Find  $P(\bar{x} > 9)$ .
  - (d) **Interpretation** Would it be unusual for a random sample of size 64 from the  $x$  distribution to have a sample mean greater than 9? Explain.

## ◀ Basic Computation Problems

These problems focus student attention on relevant formulas, requirements, and computational procedures. After practicing these skills, students are more confident as they approach real-world applications.

## Expand Your Knowledge Problems ▶

Expand Your Knowledge problems present optional enrichment topics that go beyond the material introduced in a section. Vocabulary and concepts needed to solve the problems are included at point-of-use, expanding students' statistical literacy.



30. **Expand Your Knowledge: Geometric Mean** When data consist of percentages, ratios, growth rates, or other rates of change, the *geometric mean* is a useful measure of central tendency. For  $n$  data values,

$$\text{Geometric mean} = \sqrt[n]{\text{product of the } n \text{ data values}}, \text{ assuming all data values are positive}$$

To find the *average growth factor* over 5 years of an investment in a mutual fund with growth rates of 10% the first year, 12% the second year, 14.8% the third year, 3.8% the fourth year, and 6% the fifth year, take the geometric mean of 1.10, 1.12, 1.148, 1.038, and 1.16. Find the average growth factor of this investment.

Note that for the same data, the relationships among the harmonic, geometric, and arithmetic means are harmonic mean  $\leq$  geometric mean  $\leq$  arithmetic mean (Source: *Oxford Dictionary of Statistics*).

# DIRECTION AND PURPOSE

Real knowledge is delivered through direction, not just facts. *Understandable Statistics: Concepts and Methods* ensures the student knows what is being covered and why at every step along the way to statistical literacy.

## Chapter Preview Questions

Preview Questions at the beginning of each chapter give the student a taste of what types of questions can be answered with an understanding of the knowledge to come.

## NORMAL CURVES AND SAMPLING DISTRIBUTIONS

### PREVIEW QUESTIONS

- What are some characteristics of a normal distribution? What does the empirical rule tell you about data spread around the mean? How can this information be used in quality control? (SECTION 6.1)
- Can you compare apples and oranges, or maybe elephants and butterflies? In most cases, the answer is no—unless you first standardize your measurements. What are a standard normal distribution and a standard  $z$  score? (SECTION 6.2)
- How do you convert any normal distribution to a standard normal distribution? How do you find probabilities of “standardized events”? (SECTION 6.3)
- As humans, our experiences are finite and limited. Consequently, most of the important decisions in our lives are based on sample (incomplete) information. What is a probability sampling distribution? How will sampling distributions help us make good decisions based on incomplete information? (SECTION 6.4)
- There is an old saying: All roads lead to Rome. In statistics, we could recast this saying: All probability distributions average out to be normal distributions (as the sample size increases). How can we take advantage of this in our study of sampling distributions? (SECTION 6.5)
- The binomial and normal distributions are two of the most important probability distributions in statistics. Under certain limiting conditions, the binomial can



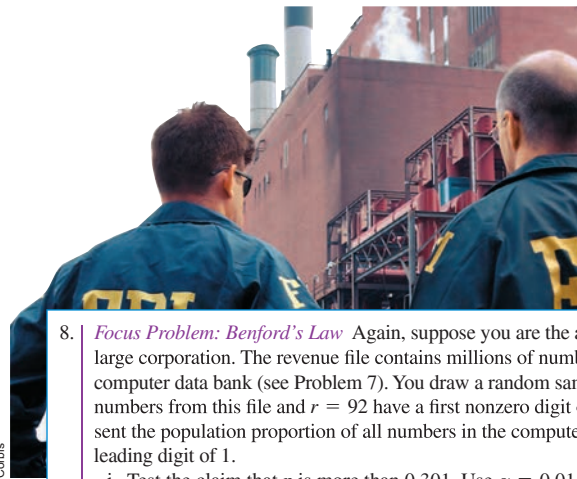
### FOCUS PROBLEM

#### Benford's Law: The Importance of Being Number 1

Benford's Law states that in a wide variety of circumstances, numbers have “1” as their first nonzero digit disproportionately often. Benford's Law applies to such diverse topics as the drainage areas of rivers; properties of chemicals; populations of towns; figures in newspapers, magazines, and government reports; and the half-lives of radioactive atoms!

Specifically, such diverse measurements begin with “1” about 30% of the time, with “2” about 18% of time, and with “3” about 12.5% of the time. Larger digits occur less often. For example, less than 5% of the numbers in circumstances such as these begin with the digit 9. This is in dramatic contrast to a random sampling situation, in which each of the digits 1 through 9 has an equal chance of appearing.

The first nonzero digits of numbers taken from large bodies of numerical records such



8. **Focus Problem: Benford's Law** Again, suppose you are the auditor for a very large corporation. The revenue file contains millions of numbers in a large computer data bank (see Problem 7). You draw a random sample of  $n = 228$  numbers from this file and  $r = 92$  have a first nonzero digit of 1. Let  $p$  represent the population proportion of all numbers in the computer file that have a leading digit of 1.
- Test the claim that  $p$  is more than 0.301. Use  $\alpha = 0.01$ .
  - If  $p$  is in fact larger than 0.301, it would seem there are too many numbers in the file with leading 1's. Could this indicate that the books have been “cooked” by artificially lowering numbers in the file? Comment from the point of view of the Internal Revenue Service. Comment from the perspective of the Federal Bureau of Investigation as it looks for “profit skimming” by unscrupulous employees.
  - Comment on the following statement: “If we reject the null hypothesis at level of significance  $\alpha$ , we have not *proved*  $H_0$  to be false. We can say that the probability is  $\alpha$  that we made a mistake in rejecting  $H_0$ .” Based on the outcome of the test, would you recommend further investigation before accusing the company of fraud?

## Chapter Focus Problems

The Preview Questions in each chapter are followed by a Focus Problem, which serves as a more specific example of what questions the student will soon be able to answer. The Focus Problems are set within appropriate applications and are incorporated into the end-of-section exercises, giving students the opportunity to test their understanding.

X

# DIRECTION AND PURPOSE

## Focus Points ►

Each section opens with bulleted Focus Points describing the primary learning objectives of the section.

**SECTION 3.1**

This section can be covered quickly. Good discussion topics include *The Story of Old Faithful* in Data Highlights, Problem 1; Linking Concepts, Problem 1; and the trade winds of Hawaii (Using Technology).

Average

Mode

**Measures of Central Tendency: Mode, Median, and Mean**

**FOCUS POINTS**

- Compute mean, median, and mode from raw data.
- Interpret what mean, median, and mode tell you.
- Explain how mean, median, and mode can be affected by extreme data values.
- What is a trimmed mean? How do you compute it?
- Compute a weighted average.

The average price of an ounce of gold is \$1350. The Zippy car averages 39 miles per gallon on the highway. A survey showed the average shoe size for women is size 9.

In each of the preceding statements, *one* number is used to describe the entire sample or population. Such a number is called an *average*. There are many ways to compute averages, but we will study only three of the major ones.

The easiest average to compute is the *mode*.

The **mode** of a data set is the value that occurs most frequently. *Note:* If a data set has no single value that occurs more frequently than any other, then that data set has no mode.

**EXAMPLE 1**

**MODE**

Count the letters in each word of this sentence and give the mode. The numbers of letters in the words of the sentence are


5 3 7 2 4 4 2 4 8 3 4 3 4

## LOOKING FORWARD

In later chapters we will use information based on a sample and sample statistics to estimate population parameters (Chapter 7) or make decisions about the value of population parameters (Chapter 8).

## ◀ Looking Forward

This feature shows students where the presented material will be used later. It helps motivate students to pay a little extra attention to key topics.



## CHAPTER REVIEW

SUMMARY

In this chapter, you've seen that statistics is the study of how to collect, organize, analyze, and interpret numerical information from populations or samples. This chapter discussed some of the features of data and ways to collect data. In particular, the chapter discussed

- Individuals or subjects of a study and the variables associated with those individuals
- Data classification as qualitative or quantitative, and levels of measurement of data
- Sample and population data. Summary measurements from sample data are called statistics, and those from populations are called parameters.

- Sampling strategies, including simple random, stratified, systematic, multistage, and convenience. Inferential techniques presented in this text are based on simple random samples.
- Methods of obtaining data: Use of a census, simulation, observational studies, experiments, and surveys
- Concerns: Undercoverage of a population, nonresponse, bias in data from surveys and other factors, effects of confounding or lurking variables on other variables, generalization of study results beyond the population of the study, and study sponsorship

## ▲ Chapter Summaries

The Summary within each Chapter Review feature now also appears in bulleted form, so students can see what they need to know at a glance.



# REAL-WORLD SKILLS

Statistics is not done in a vacuum. *Understandable Statistics: Concepts and Methods* gives students valuable skills for the real world with technology instruction, genuine applications, actual data, and group projects.

## REVISED! Tech Notes ►

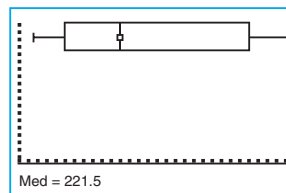
Tech Notes appearing throughout the text give students helpful hints on using TI-84Plus and TI-nspire (with TI-84Plus keypad) and TI-83Plus calculators, Microsoft Excel 2010, and Minitab to solve a problem. They include display screens to help students visualize and better understand the solution.



### TECH NOTES *Box-and-Whisker Plot*

Both Minitab and the TI-84Plus/TI-83Plus/TI-nspire calculators support box-and-whisker plots. On the TI-84Plus/TI-83Plus/TI-nspire, the quartiles  $Q_1$  and  $Q_3$  are calculated as we calculate them in this text. In Minitab and Excel 2010, they are calculated using a slightly different process.

**TI-84Plus/TI-83Plus/TI-nspire (with TI-84Plus Keypad)** Press **STATPLOT** ► **On**. Highlight box plot. Use **Trace** and the arrow keys to display the values of the five-number summary. The display shows the plot for calories in ice cream bars.



box-and-whisker plot. However, each value of the und. On the **Home** ribbon, click the **Insert Function Statistical** as the category and scroll to **Quartile**. In location and then enter the number of the value you e quartile box for the first quartile.

t. In the dialogue box, set Display to **IQRRange Box**.



## USING TECHNOLOGY

### Binomial Distributions

Although tables of binomial probabilities can be found in most libraries, such tables are often inadequate. Either the value of  $p$  (the probability of success on a trial) you are looking for is not in the table, or the value of  $n$  (the number of trials) you are looking for is too large for the table. In Chapter 6, we will study the normal approximation to the binomial. This approximation is a great help in many practical applications. Even so, we sometimes use the formula for the binomial probability distribution on a computer or graphing calculator to compute the probability we want.

### Applications

The following percentages were obtained over many years of observation by the U.S. Weather Bureau. All data listed are for the month of December.

Location	Long-Term Mean % of Clear Days in Dec.
Juneau, Alaska	18%
Seattle, Washington	24%
Hilo, Hawaii	36%
Honolulu, Hawaii	60%
Las Vegas, Nevada	75%
Phoenix, Arizona	77%

Adapted from *Local Climatological Data*, U.S. Weather Bureau publication, "Normals, Means, and Extremes" Table.

In the locations listed, the month of December is a relatively stable month with respect to weather. Since weather patterns from one day to the next are more or less the same, it is reasonable to use a binomial probability model.

- Let  $r$  be the number of clear days in December. Since December has 31 days,  $0 \leq r \leq 31$ . Using appropriate computer software or calculators available to you, find the probability  $P(r)$  for each of the listed locations when  $r = 0, 1, 2, \dots, 31$ .
- For each location, what is the expected value of the probability distribution? What is the standard deviation?

You may find that using cumulative probabilities and appropriate subtraction of probabilities, rather than addition of probabilities, will make finding the solutions to Applications 3 to 7 easier.

- Estimate the probability that Juneau will have at most 7 clear days in December.
- Estimate the probability that Seattle will have from 5 to 10 (including 5 and 10) clear days in December.
- Estimate the probability that Hilo will have at least 12 clear days in December.
- Estimate the probability that Phoenix will have 20 or more clear days in December.
- Estimate the probability that Las Vegas will have from 20 to 25 (including 20 and 25) clear days in December.

### Technology Hints

**TI-84Plus/TI-83Plus/TI-nspire (with TI-84 Plus keypad), Excel 2010, Minitab**

The Tech Note in Section 5.2 gives specific instructions for binomial distribution functions on the TI-84Plus/TI-83Plus/TI-nspire (with TI-84Plus keypad) calculators, Excel 2010, and Minitab.

### SPSS

In SPSS, the function **PDF.BINOM(q,n,p)** gives the probability of  $q$  successes out of  $n$  trials, where  $p$  is the probability of success on a single trial. In the data editor, name a variable  $r$  and enter values 0 through  $n$ . Name another variable  $\text{Prob}_r$ . Then use the menu choices **Transform ► Compute**. In the dialogue box, use  $\text{Prob}_r$  for the target variable. In the function group, select **PDF and Noncentral PDF**. In the function box, select **PDE.BINOM(q,n,p)**. Use the variable  $r$  for  $q$  and appropriate values for  $n$  and  $p$ . Note that the function **CDF.BINOM(q,n,p)**, from the **CDF and Noncentral CDF** group, gives the cumulative probability of 0 through  $q$  successes.

## ◀ REVISED! Using Technology

Further technology instruction is available at the end of each chapter in the Using Technology section. Problems are presented with real-world data from a variety of disciplines that can be solved by using TI-84Plus and TI-nspire (with TI-84Plus keypad) and TI-83Plus calculators, Microsoft Excel 2010, and Minitab.

# REAL-WORLD SKILLS

## EXAMPLE 13

### CENTRAL LIMIT THEOREM

A certain strain of bacteria occurs in all raw milk. Let  $x$  be the bacteria count per milliliter of milk. The health department has found that if the milk is not contaminated, then  $x$  has a distribution that is more or less mound-shaped and symmetrical. The mean of the  $x$  distribution is  $\mu = 2500$ , and the standard deviation is  $\sigma = 300$ . In a large commercial dairy, the health inspector takes 42 random samples of the milk produced each day. At the end of the day, the bacteria count in each of the 42 samples is averaged to obtain the sample mean bacteria count  $\bar{x}$ .

(a) Assuming the milk is not contaminated, what is the distribution of  $\bar{x}$ ?

**SOLUTION:** The sample size is  $n = 42$ . Since this value exceeds 30, the central limit theorem applies, and we know the distribution of  $\bar{x}$  is normal with mean and standard deviation

### ◀ UPDATED! Applications

Real-world applications are used from the beginning to introduce each statistical process. Rather than just crunching numbers, students come to appreciate the value of statistics through relevant examples.

Most exercises in each section ▶ are applications problems.

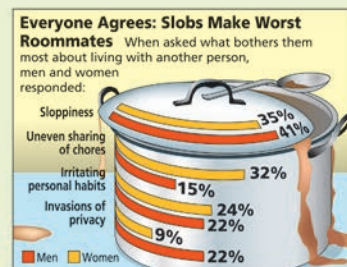
11. **Pain Management: Laser Therapy** “Effect of Helium-Neon Laser Auriculotherapy on Experimental Pain Threshold” is the title of an article in the journal *Physical Therapy* (Vol. 70, No. 1, pp. 24–30). In this article, laser therapy was discussed as a useful alternative to drugs in pain management of chronically ill patients. To measure pain threshold, a machine was used that delivered low-voltage direct current to different parts of the body (wrist, neck, and back). The machine measured current in milliamperes (mA). The pretreatment experimental group in the study had an average threshold of pain (pain was first detectable) at  $\mu = 3.15$  mA with standard deviation  $\sigma = 1.45$  mA. Assume that the distribution of threshold pain, measured in milliamperes, is symmetrical and more or less mound-shaped. Use the empirical rule to
  - (a) estimate a range of milliamperes centered about the mean in which about 68% of the experimental group had a threshold of pain.
  - (b) estimate a range of milliamperes centered about the mean in which about 95% of the experimental group had a threshold of pain.
12. **Control Charts: Yellowstone National Park** Yellowstone Park Medical Services (YPMS) provides emergency health care for park visitors. Such health care includes treatment for everything from indigestion and sunburn to more serious injuries. A recent issue of *Yellowstone Today* (National Park

### DATA HIGHLIGHTS: GROUP PROJECTS

Break into small groups and discuss the following topics. Organize a brief outline in which you summarize the main points of your group discussion.

1. Examine Figure 2-20, “Everyone Agrees: Slobs Make Worst Roommates.” This is a clustered bar graph because two percentages are given for each response category: responses from men and responses from women. Comment about how the artistic rendition has slightly changed the format of a bar graph. Do the bars seem to have lengths that accurately reflect the relative percentages of the responses? In your own opinion, does the artistic rendition enhance or confuse the information? Explain. Which characteristic of “worst roommates” does the graphic seem to illustrate? Can this graph be considered a Pareto chart for men? for women? Why or why not? From the information given in the figure, do you think the survey just listed the four given annoying characteristics? Do you think a respondent could choose more than one characteristic? Explain

FIGURE 2-20



Source: Advantage Business Research for Mattel Compatibility

### ◀ Data Highlights: Group Projects

Using Group Projects, students gain experience working with others by discussing a topic, analyzing data, and collaborating to formulate their response to the questions posed in the exercise.

# MAKING THE JUMP

Get to the “Aha!” moment faster. *Understandable Statistics: Concepts and Methods* provides the push students need to get there through guidance and example.

## PROCEDURE

### HOW TO TEST $\mu$ WHEN $\sigma$ IS KNOWN

#### Requirements

Let  $x$  be a random variable appropriate to your application. Obtain a simple random sample (of size  $n$ ) of  $x$  values from which you compute the sample mean  $\bar{x}$ . The value of  $\sigma$  is already known (perhaps from a previous study). If you can assume that  $x$  has a normal distribution, then any sample size  $n$  will work. If you cannot assume this, then use a sample size  $n \geq 30$ .

#### Procedure

1. In the context of the application, state the *null and alternate hypotheses* and set the *level of significance*  $\alpha$ .
2. Use the known  $\sigma$ , the sample size  $n$ , the value of  $x$  from the sample, and  $\mu$  from the null hypothesis to compute the standardized *sample test statistic*.

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

3. Use the standard normal or two-tailed, to find the
4. *Conclude* the test. If  $P$ -value do not reject  $H_0$ .
5. *Interpret your conclusion*

## ◀ Procedures and Requirements

Procedure display boxes summarize simple step-by-step strategies for carrying out statistical procedures and methods as they are introduced. Requirements for using the procedures are also stated. Students can refer back to these boxes as they practice using the procedures.

### GUIDED EXERCISE 11

### PROBABILITY REGARDING $\bar{x}$

In mountain country, major highways sometimes use tunnels instead of long, winding roads over high passes. However, too many vehicles in a tunnel at the same time can cause a hazardous situation. Traffic engineers are studying a long tunnel in Colorado. If  $x$  represents the time for a vehicle to go through the tunnel, it is known that the  $x$  distribution has mean  $\mu = 12.1$  minutes and standard deviation  $\sigma = 3.8$  minutes under ordinary traffic conditions. From a histogram of  $x$  values, it was found that the  $x$  distribution is mound-shaped with some symmetry about the mean.

Engineers have calculated that, *on average*, vehicles should spend from 11 to 13 minutes in the tunnel. If the time is less than 11 minutes, traffic is moving too fast for safe travel in the tunnel. If the time is more than 13 minutes, there is a problem of bad air quality (too much carbon monoxide and other pollutants).

Under ordinary conditions, there are about 50 vehicles in the tunnel at one time. What is the probability that the mean time for 50 vehicles in the tunnel will be from 11 to 13 minutes? We will answer this question in steps.

- (a) Let  $\bar{x}$  represent the sample mean based on samples of size 50. Describe the  $\bar{x}$  distribution.

➔ From the central limit theorem, we expect the  $\bar{x}$  distribution to be approximately normal, with mean and standard deviation

$$\mu_{\bar{x}} = \mu = 12.1 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.8}{\sqrt{50}} \approx 0.54$$

- (b) Find  $P(11 < \bar{x} < 13)$ .



Jupiter Images

➔ We convert the interval  $11 < \bar{x} < 13$  to a standard  $z$  interval and use the standard normal probability table to find our answer. Since

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \approx \frac{\bar{x} - 12.1}{0.54}$$

$$\bar{x} = 11 \text{ converts to } z \approx \frac{11 - 12.1}{0.54} = -2.04$$

$$\text{and } \bar{x} = 13 \text{ converts to } z \approx \frac{13 - 12.1}{0.54} = 1.67$$

Therefore,

$$P(11 < \bar{x} < 13) = P(-2.04 < z < 1.67) = 0.9525 - 0.0207 = 0.9318$$

- (c) *Interpret* your answer to part (b).

➔ It seems that about 93% of the time, there should be no safety hazard for average traffic flow.

## Guided Exercises ▶

Students gain experience with new procedures and methods through Guided Exercises. Beside each problem in a Guided Exercise, a completely worked-out solution appears for immediate reinforcement.



## Preface

Welcome to the exciting world of statistics! We have written this text to make statistics accessible to everyone, including those with a limited mathematics background. Statistics affects all aspects of our lives. Whether we are testing new medical devices or determining what will entertain us, applications of statistics are so numerous that, in a sense, we are limited only by our own imagination in discovering new uses for statistics.

## OVERVIEW

The eleventh edition of *Understandable Statistics: Concepts and Methods* continues to emphasize concepts of statistics. Statistical methods are carefully presented with a focus on understanding both the *suitability of the method* and the *meaning of the result*. Statistical methods and measurements are developed in the context of applications.

Critical thinking and interpretation are essential in understanding and evaluating information. Statistical literacy is fundamental for applying and comprehending statistical results. In this edition we have expanded and highlighted the treatment of statistical literacy, critical thinking, and interpretation.

We have retained and expanded features that made the first 10 editions of the text very readable. Definition boxes highlight important terms. Procedure displays summarize steps for analyzing data. Examples, exercises, and problems touch on applications appropriate to a broad range of interests.

The eleventh edition continues to have extensive online support. Online homework powered by a choice of Enhanced WebAssign or Aplia™ is now available through CengageBrain.com. Instructional videos are available on DVD. The companion web site at <http://www.cengage.com/statistics/brase11e> contains more than 100 data sets (in Microsoft Excel, Minitab, SPSS, and TI-84Plus/TI-83Plus/TI-*n*spire with TI-84Plus keypad ASCII file formats), technology guides, lecture aids, a glossary, and statistical tables.



New for the eleventh edition, available via Aplia™, is MindTap™ Reader, Cengage Learning's next-generation eBook. MindTap Reader provides robust opportunities for students to annotate, take notes, navigate, and interact with the text (e.g., ReadSpeaker). Annotations captured in MindTap Reader are automatically tied to the Notepad app, where they can be viewed chronologically and in a cogent, linear fashion. Instructors also can edit the text and assets in the Reader as well as add videos or URLs. Go to <http://www.cengage.com/mindtap> for more information.

## MAJOR CHANGES IN THE ELEVENTH EDITION

With each new edition, the authors reevaluate the scope, appropriateness, and effectiveness of the text's presentation and reflect on extensive user feedback. Revisions have been made throughout the text to clarify explanations of important concepts and to update problems.

### Critical Thinking, Interpretation, and Statistical Literacy

The eleventh edition of this text continues and expands the emphasis on critical thinking, interpretation, and statistical literacy. Calculators and computers are very good at providing numerical results of statistical processes. However, numbers from

a computer or calculator display are meaningless unless the user knows how to interpret the results and if the statistical process is appropriate. This text helps students determine whether or not a statistical method or process is appropriate. It helps students understand what a statistic measures. It helps students interpret the results of a confidence interval, hypothesis test, or linear regression model.

### New Interpretation Features

To further understanding and interpretation of statistical concepts, methods, and results, we have included two new special features: **What Does (a concept, method, or result) Tell Us?** and **Important Features of a (concept, method, or result)**. These features summarize the information we obtain from concepts and statistical processes and give additional insights for further application.



### New Expand Your Knowledge Problems and Quick Overview Topics With Additional Applications

*Expand Your Knowledge problems* do just that! These are optional but contain very useful information taken from the vast literature of statistics. These topics are not included in the main text but are easily learned using material from the section or previous sections. Although these topics are optional, the authors feel they add depth and enrich a student's learning experience. Each topic was chosen for its relatively straightforward presentation and useful applications. All such problems and their applications are flagged with a sun logo.

New *Expand Your Knowledge problems* in the eleventh edition involve donut graphs; stratified sampling and the best estimate for the population mean  $\mu$ ; the process of using minimal variance for linear combinations of independent random variables; and serial correlation (also called autocorrelation).

Some of the other topics in *Expand Your Knowledge problems* or quick overviews include graphs such as dotplots and variations on stem-and-leaf plots; outliers in stem-and-leaf plots; harmonic and geometric means; moving averages; calculating odds in favor and odds against; extension of conditional probability to various distributions such as the Poisson distribution and the normal distribution; Bayes's theorem; additional probability distributions such as the multinomial distribution, negative binomial distribution, hypergeometric distribution, continuous uniform distribution, and exponential distribution; waiting time between Poisson events; quick estimate of the standard deviation using the Empirical rule; plus four confidence intervals for proportions; Satterthwaite's approximation for degrees of freedom in confidence intervals and hypothesis tests; relationship between confidence intervals and two-tailed hypothesis testing; pooled two-sample procedures for confidence intervals and hypothesis tests; resampling (also known as bootstrap); simulations of confidence intervals and hypothesis tests using different samples of the same size; mean and standard deviation for linear combinations of dependent random variables; logarithmic transformations with the exponential growth model and the power law model; and polynomial (curvilinear) regression.

For location of these optional topics in the text, please see the index.

### Revised Examples and New Section Problems

Examples and guided exercises have been updated and revised. Additional section problems emphasize critical thinking and interpretation of statistical results.

### Excel 2010 and Most Recent Operating System for the TI-84Plus/TI-83Plus Calculators

Excel 2010 instructions are included in the *Tech Notes* and *Using Technology*. The latest operating system (v2.55MP) for the TI-84Plus/TI-83Plus calculators is also

discussed, with new functions such as the inverse  $t$  distribution and the chi-square goodness of fit test described. One convenient feature of the operating system is that it provides on-screen prompts for inputs required for many probability and statistical functions. This operating system is already on new TI-84Plus/TI-83Plus calculators and is available for download to older calculators at the Texas Instruments web site.

### Revised Electronic Student Resources

Digital student resources and online tools that accompany *Understandable Statistics* have been revised in accordance with recommendations from both student and faculty users. Online interactive learning solutions, such as Aplia™ for Statistics—featuring the new MindTap™ Reader—and Enhanced WebAssign, are both available.

## CONTINUING CONTENT

### Introduction of Hypothesis Testing Using $P$ -Values

In keeping with the use of computer technology and standard practice in research, hypothesis testing is introduced using  $P$ -values. The critical region method is still supported but not given primary emphasis.

### Use of Student's $t$ Distribution in Confidence Intervals and Testing of Means

If the normal distribution is used in confidence intervals and testing of means, then the *population standard deviation must be known*. If the population standard deviation is not known, then under conditions described in the text, the Student's  $t$  distribution is used. This is the most commonly used procedure in statistical research. It is also used in statistical software packages such as Microsoft Excel, Minitab, SPSS, and TI-84Plus/TI-83Plus/TI-*n*spire calculators.

### Confidence Intervals and Hypothesis Tests of Difference of Means

If the normal distribution is used, then both population standard deviations must be known. When this is not the case, the Student's  $t$  distribution incorporates an approximation for  $t$ , with a commonly used conservative choice for the degrees of freedom. Satterthwaite's approximation for the degrees of freedom as used in computer software is also discussed. The pooled standard deviation is presented for appropriate applications ( $\sigma_1 \approx \sigma_2$ ).

## FEATURES IN THE ELEVENTH EDITION

### Chapter and Section Lead-ins

- *Preview Questions* at the beginning of each chapter are keyed to the sections.
- *Focus Problems* at the beginning of each chapter demonstrate types of questions students can answer once they master the concepts and skills presented in the chapter.
- *Focus Points* at the beginning of each section describe the primary learning objectives of the section.

### Carefully Developed Pedagogy

- *Examples* show students how to select and use appropriate procedures.
- *Guided Exercises* within the sections give students an opportunity to work with a new concept. Completely worked-out solutions appear beside each exercise to give immediate reinforcement.

- *Definition boxes* highlight important definitions throughout the text.
- *Procedure displays* summarize key strategies for carrying out statistical procedures and methods. Conditions required for using the procedure are also stated.
- **NEW!** *What Does (a concept method or result) Tell Us?* summarizes information we obtain from the named concepts and statistical processes and gives insight for additional application.
- **NEW!** *Important Features of a (concept, method, or result)* summarizes the features of the listed item.
- *Looking Forward* features give a brief preview of how a current topic is used later.
- *Labels* for each example or guided exercise highlight the technique, concept, or process illustrated by the example or guided exercise. In addition, labels for section and chapter problems describe the field of application and show the wide variety of subjects in which statistics is used.
- *Section and chapter problems* require the student to use all the new concepts mastered in the section or chapter. Problem sets include a variety of real-world applications with data or settings from identifiable sources. Key steps and solutions to odd-numbered problems appear at the end of the book.
- *Basic Computation problems* ask students to practice using formulas and statistical methods on very small data sets. Such practice helps students understand what a statistic measures.
- *Statistical Literacy problems* ask students to focus on correct terminology and processes of appropriate statistical methods. Such problems occur in every section and chapter problem set.
- *Interpretation problems* ask students to explain the meaning of the statistical results in the context of the application.
- *Critical Thinking problems* ask students to analyze and comment on various issues that arise in the application of statistical methods and in the interpretation of results. These problems occur in every section and chapter problem set.
- *Expand Your Knowledge problems* present enrichment topics such as negative binomial distribution; conditional probability utilizing binomial, Poisson, and normal distributions; estimation of standard deviation from a range of data values; and more.
- *Cumulative review problem sets* occur after every third chapter and include key topics from previous chapters. Answers to *all* cumulative review problems are given at the end of the book.
- *Data Highlights and Linking Concepts* provide group projects and writing projects.
- *Viewpoints* are brief essays presenting diverse situations in which statistics is used.
- *Design and photos* are appealing and enhance readability.

### Technology Within the Text

- *Tech Notes* within sections provide brief point-of-use instructions for the TI-84Plus, TI-83Plus, and TI-*n*spire (with 84Plus keypad) calculators, Microsoft Excel 2010, and Minitab.
- *Using Technology* sections show the use of SPSS as well as the TI-84Plus, TI-83Plus, and TI-*n*spire (with TI-84Plus keypad) calculators, Microsoft Excel, and Minitab.

## ALTERNATE ROUTES THROUGH THE TEXT

*Understandable Statistics: Concepts and Methods*, Eleventh Edition, is designed to be flexible. It offers the professor a choice of teaching possibilities. In most one-semester courses, it is not practical to cover all the material in depth. However, depending on the emphasis of the course, the professor may choose to cover various topics. For help in topic selection, refer to the Table of Prerequisite Material on page 1.

- *Introducing linear regression early.* For courses requiring an early presentation of linear regression, the descriptive components of linear regression (Sections 9.1

and 9.2) can be presented any time after Chapter 3. However, inference topics involving predictions, the correlation coefficient  $\rho$ , and the slope of the least-squares line  $\beta$  require an introduction to confidence intervals (Sections 7.1 and 7.2) and hypothesis testing (Sections 8.1 and 8.2).

- *Probability*. For courses requiring minimal probability, Section 4.1 (What Is Probability?) and the first part of Section 4.2 (Some Probability Rules—Compound Events) will be sufficient.

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## Additional Resources — Get More from Your Textbook!



### MindTap™

Available via Aplia™ is MindTap™ Reader, Cengage Learning's next-generation eBook. An integral component of the MindTap environment, MindTap Reader replaces the traditional "e-book" and provides robust opportunities for students to annotate, take notes, navigate, and interact with the text (e.g., ReadSpeaker). Annotations captured in MindTap Reader are automatically tied to the Notepad app, where they can be viewed chronologically and in a cogent, linear fashion. Instructors also can edit the text and assets in the Reader, as well as add videos or URLs.

Go to <http://www.cengage.com/mindtap> for more information.

### Instructor Resources

**Annotated Instructor's Edition (AIE)** Answers to all exercises, teaching comments, and pedagogical suggestions appear in the margin, or at the end of the text in the case of large graphs.

**Solution Builder** Contains complete solutions to all exercises in the text, including those in the Chapter Review and Cumulative Review Problems in online format. Solution Builder allows instructors to create customized, secure PDF printouts of solutions matched exactly to the exercises assigned for class. Available to adoptions by signing up at [www.cengage.com/solutionbuilder](http://www.cengage.com/solutionbuilder).

**Cengage Learning Testing Powered by Cognero** A flexible, online system that allows you to:

- author, edit, and manage test bank content from multiple Cengage Learning solutions
- create multiple test versions in an instant
- deliver tests from your LMS, your classroom or wherever you want

**Companion Website** The companion website at <http://www.cengage.com/brase> contains a variety of resources.

- Microsoft® PowerPoint® lecture slides
- Figures from the book
- More than 100 data sets in a variety of formats, including
  - Microsoft Excel
  - Minitab
  - SPSS
  - TI-84Plus/TI-83Plus/TI-nspire with 84plus keypad ASCII file formats
- Technology guides for the following programs
  - TI-84Plus, TI-83Plus, and TI-nspire graphing calculators
  - Minitab software (version 14)
  - Microsoft Excel (2010/2007)
  - SPSS Statistics software
- Lecture aids like Teaching Hints and Frequently Used Formulas
- Statistical tables and a glossary

### Student Resources

**Student Solutions Manual** Provides solutions to the odd-numbered section and chapter exercises and to all the Cumulative Review exercises in the student textbook.

**Instructional DVDs** Hosted by Dana Mosely, these text-specific DVDs cover all sections of the text and provide explanations of key concepts, examples, exercises, and applications in a lecture-based format. DVDs are close-captioned for the hearing-impaired.



Aplia is an online interactive learning solution that helps students improve comprehension—and their grade—by integrating a variety of mediums and tools such as video, tutorials, practice tests, and an interactive eBook. Created by a professor to enhance his own courses, Aplia provides automatically graded assignments with detailed, immediate feedback on every question, and innovative teaching materials. More than 1,000,000 students have used Aplia at over 1,800 institutions.



JMP is a statistics software for Windows and Macintosh computers from SAS, the market leader in analytics software and services for industry. JMP Student Edition is a streamlined, easy-to-use version that provides all the statistical analysis and graphics covered in this textbook. Once data is imported, students will find that most procedures require just two or three mouse clicks. JMP can import data from a variety of formats, including Excel and other statistical packages, and you can easily copy and paste graphs and output into documents.

JMP also provides an interface to explore data visually and interactively, which will help your students develop a healthy relationship with their data, work more efficiently with data, and tackle difficult statistical problems more easily. Because its output provides both statistics and graphs together, the student will better see and understand the application of concepts covered in this book as well. JMP Student Edition also contains some unique platforms for student projects, such as mapping and scripting. JMP functions in the same way on both Windows and Macintosh platforms and instructions contained with this book apply to both platforms.

Access to this software is available with new copies of the book. Students can purchase JMP standalone via CengageBrain.com or [www.jmp.com/getse](http://www.jmp.com/getse).

**Minitab® and IBM SPSS** These statistical software packages manipulate and interpret data to produce textual, graphical, and tabular results. Minitab® and/or SPSS may be packaged with the textbook. Student versions are available.

The companion website at <http://www.cengage.com/statistics/brase11e> contains useful assets for students.

- **Technology Guides** Separate guides exist with information and examples for each of four technology tools. Guides are available for the TI-84Plus, TI-83Plus, and TI-*n*spire graphing calculators, Minitab software (version 14) Microsoft Excel (2010/2007), and SPSS Statistics software.
- **Interactive Teaching and Learning Tools** include glossary flashcards, online datasets (in Microsoft Excel, Minitab, SPSS, and TI-84Plus/TI-83Plus/TI-*n*spire with TI-84Plus keypad ASCII file formats), statistical tables and formulas, and more.

**Enhanced WebAssign** Offers an extensive online program for Statistics to encourage the practice that's so critical for concept mastery. The meticulously crafted pedagogy and exercises in Brase and Brase's text become even more effective in Enhanced WebAssign.

**CengageBrain.com** Provides the freedom to purchase online homework and other materials à la carte exactly what you need, when you need it.

For more information, visit <http://www.cengage.com/statistics/brase11e> or contact your local Cengage Learning sales representative.



## Table of Prerequisite Material

Chapter	Prerequisite Sections
1 Getting Started	None
2 Organizing Data	1.1, 1.2
3 Averages and Variation	1.1, 1.2, 2.1
4 Elementary Probability Theory	1.1, 1.2, 2.1, 3.1, 3.2
5 The Binomial Probability Distribution and Related Topics	1.1, 1.2, 2.1, 3.1, 3.2, 4.1, 4.2 4.3 useful but not essential
6 Normal Curves and Sampling Distributions (omit 6.6) (include 6.6)	1.1, 1.2, 2.1, 3.1, 3.2, 4.1, 4.2, 5.1 also 5.2, 5.3
7 Estimation (omit 7.3 and parts of 7.4) (include 7.3 and all of 7.4)	1.1, 1.2, 2.1, 3.1, 3.2, 4.1, 4.2, 5.1, 6.1, 6.2, 6.3, 6.4, 6.5 also 5.2, 5.3, 6.6
8 Hypothesis Testing (omit 8.3 and part of 8.5) (include 8.3 and all of 8.5)	1.1, 1.2, 2.1, 3.1, 3.2, 4.1, 4.2, 5.1, 6.1, 6.2, 6.3, 6.4, 6.5 also 5.2, 5.3, 6.6
9 Correlation and Regression (9.1 and 9.2) (9.3 and 9.4)	1.1, 1.2, 3.1, 3.2 also 4.1, 4.2, 5.1, 6.1, 6.2, 6.3, 6.4, 6.5, 7.1, 7.2, 8.1, 8.2
10 Chi-Square and $F$ Distributions (omit 10.3) (include 10.3)	1.1, 1.2, 2.1, 3.1, 3.2, 4.1, 4.2, 5.1, 6.1, 6.2, 6.3, 6.4, 6.5, 8.1 also 7.1
11 Nonparametric Statistics	1.1, 1.2, 2.1, 3.1, 3.2, 4.1, 4.2, 5.1, 6.1, 6.2, 6.3, 6.4, 6.5, 8.1, 8.3

# 1

MLB Photos/Getty Images Sport/Getty Images



- 1.1 What Is Statistics?
- 1.2 Random Samples
- 1.3 Introduction to Experimental Design

Louis Pasteur (1822–1895) is the founder of modern bacteriology. At age 57, Pasteur was studying cholera. He accidentally left some bacillus culture unattended in his laboratory during the summer. In the fall, he injected laboratory animals with this bacilli. To his surprise, the animals did not die—in fact, they thrived and were resistant to cholera.

When the final results were examined, it is said that Pasteur remained silent for a minute and then exclaimed, as if he had seen a vision, “Don’t you see they have been vaccinated!” Pasteur’s work ultimately saved many human lives.

Most of the important decisions in life involve incomplete information. Such decisions often involve so many complicated factors that a complete analysis is not practical or even possible. We are often forced into the position of making a guess based on limited information.

As the first quote reminds us, our chances of success are greatly improved if we have a “prepared mind.” The statistical methods you will learn in this book will help you achieve a prepared mind for the study of many different fields. The second quote reminds us that statistics is an important tool, but it is not a replacement for an in-depth knowledge of the field to which it is being applied.

The authors of this book want you to understand and enjoy statistics. The reading material will *tell you* about the subject. The examples will *show you* how it works. To understand, however, you must *get involved*. Guided exercises, calculator and computer applications, section and chapter problems, and writing exercises are all designed to get you involved in the subject. As you grow in your understanding of statistics, we believe you will enjoy learning a subject that has a world full of interesting applications.

*Chance favors the prepared mind.*

—LOUIS PASTEUR

*Statistical techniques are tools of thought . . . not substitutes for thought.*

—ABRAHAM KAPLAN



Bettmann/Corbis

# GETTING STARTED

## PREVIEW QUESTIONS

- Why is statistics important? (SECTION 1.1)
- What is the nature of data? (SECTION 1.1)
- How can you draw a random sample? (SECTION 1.2)
- What are other sampling techniques? (SECTION 1.2)
- How can you design ways to collect data? (SECTION 1.3)

## FOCUS PROBLEM

### Where Have All the Fireflies Gone?

A feature article in *The Wall Street Journal* discusses the disappearance of fireflies. In the article, Professor Sara Lewis of Tufts University and other scholars express concern about the decline in the worldwide population of fireflies.

There are a number of possible explanations for the decline, including habitat reduction of woodlands, wetlands, and open fields; pesticides; and pollution. Artificial nighttime lighting might interfere with the Morse-code-like mating ritual of the fireflies. Some chemical companies pay a bounty for fireflies because the insects contain two rare chemicals used in medical research and electronic detection systems used in spacecraft.

What does any of this have to do with statistics?

The truth, at this time, is that no one really knows (a) how much the world firefly population has declined or (b) how to explain the decline. The population of all fireflies is simply too large to study in its entirety.

In any study of fireflies, we must rely on incomplete information from samples. Furthermore, from these samples we must draw realistic conclusions that have statistical integrity. This is the kind of work that makes use of statistical methods to determine ways to collect, analyze, and investigate data.

Suppose you are conducting a study to compare firefly populations exposed to normal daylight/darkness conditions with firefly populations exposed to continuous light (24 hours a day). You set up two firefly colonies in a laboratory environment. The two colonies are identical except that one colony is exposed to normal daylight/darkness conditions and the other is exposed to continuous light. Each colony is populated with the same number of mature fireflies. After 72 hours, you count the number of living fireflies in each colony.

After completing this chapter, you will be able to answer the following questions.

- (a) Is this an experiment or an observation study? Explain.
- (b) Is there a control group? Is there a treatment group?



STUDIO SATO/amainimages/Corbis



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Adapted from Ohio State University Firefly Files logo

For online student resources, visit the Brase/Brase, *Understandable Statistics*, 11th edition web site at <http://www.cengage.com/statistics/brase11e>

- (c) What is the variable in this study?
- (d) What is the level of measurement (nominal, interval, ordinal, or ratio) of the variable? (See Problem 11 of the Chapter 1 Review Problems.)

## SECTION 1.1

### What Is Statistics?

#### FOCUS POINTS

- Identify variables in a statistical study.
- Distinguish between quantitative and qualitative variables.
- Identify populations and samples.
- Distinguish between parameters and statistics.
- Determine the level of measurement.
- Compare descriptive and inferential statistics.

#### INTRODUCTION

Decision making is an important aspect of our lives. We make decisions based on the information we have, our attitudes, and our values. Statistical methods help us examine information. Moreover, statistics can be used for making decisions when we are faced with uncertainties. For instance, if we wish to estimate the proportion of people who will have a severe reaction to a flu shot without giving the shot to everyone who wants it, statistics provides appropriate methods. Statistical methods enable us to look at information from a small collection of people or items and make inferences about a larger collection of people or items.

Procedures for analyzing data, together with rules of inference, are central topics in the study of statistics.

#### Statistics

**Statistics** is the study of how to collect, organize, analyze, and interpret numerical information from data.

The subject of statistics is multifaceted. The following definition of statistics is found in the *International Encyclopedia of Statistical Science*, edited by Miodrag Lovric. Professor David Hand of Imperial College London—the president of the Royal Statistical Society—presents the definition in his article “Statistics: An Overview.”

**Statistics** is both the science of uncertainty and the technology of extracting information from data.

The statistical procedures you will learn in this book should supplement your built-in system of inference—that is, the results from statistical procedures and good sense should dovetail. Of course, statistical methods themselves have no power to work miracles. These methods can help us make some decisions, but not all conceivable decisions. Remember, even a properly applied statistical procedure is no more accurate than the data, or facts, on which it is based. Finally, statistical results should be interpreted by one who understands not only the methods, but also the subject matter to which they have been applied.

The general prerequisite for statistical decision making is the gathering of data. First, we need to identify the individuals or objects to be included in the study and the characteristics or features of the individuals that are of interest.

### Individuals Variable

**Individuals** are the people or objects included in the study.  
A **variable** is a characteristic of the individual to be measured or observed.

For instance, if we want to do a study about the people who have climbed Mt. Everest, then the individuals in the study are all people who have actually made it to the summit. One variable might be the height of such individuals. Other variables might be age, weight, gender, nationality, income, and so on. Regardless of the variables we use, we would not include measurements or observations from people who have not climbed the mountain.

The variables in a study may be *quantitative* or *qualitative* in nature.

### Quantitative variable Qualitative variable

A **quantitative variable** has a value or numerical measurement for which operations such as addition or averaging make sense. A **qualitative variable** describes an individual by placing the individual into a category or group, such as male or female.

For the Mt. Everest climbers, variables such as height, weight, age, or income are *quantitative* variables. *Qualitative variables* involve nonnumerical observations such as gender or nationality. Sometimes qualitative variables are referred to as *categorical variables*.

### Categorical variable

Another important issue regarding data is their source. Do the data comprise information from *all* individuals of interest, or from just *some* of the individuals?

### Population data Sample data

In **population data**, the data are from *every* individual of interest.  
In **sample data**, the data are from *only some* of the individuals of interest.

It is important to know whether the data are population data or sample data. Data from a specific population are fixed and complete. Data from a sample may vary from sample to sample and are *not* complete.

### Population parameter

A **population parameter** is a numerical measure that describes an aspect of a population.

### Sample statistic

A **sample statistic** is a numerical measure that describes an aspect of a sample.

For instance, if we have data from *all* the individuals who have climbed Mt. Everest, then we have population data. The proportion of males in the *population* of all climbers who have conquered Mt. Everest is an example of a *parameter*.

On the other hand, if our data come from just some of the climbers, we have sample data. The proportion of male climbers in the *sample* is an example of a *statistic*. Note that different samples may have different values for the proportion of male climbers. One of the important features of sample statistics is that they can vary from sample to sample, whereas population parameters are fixed for a given population.

## LOOKING FORWARD

In later chapters we will use information based on a sample and sample statistics to estimate population parameters (Chapter 7) or make decisions about the value of population parameters (Chapter 8).

## EXAMPLE 1

### USING BASIC TERMINOLOGY

The Hawaii Department of Tropical Agriculture is conducting a study of ready-to-harvest pineapples in an experimental field.

- (a) The pineapples are the *objects* (individuals) of the study. If the researchers are interested in the individual weights of pineapples in the field, then the *variable* consists of weights. At this point, it is important to specify units of measurement





and degrees of accuracy of measurement. The weights could be measured to the nearest ounce or gram. Weight is a *quantitative* variable because it is a numerical measure. If weights of *all* the ready-to-harvest pineapples in the field are included in the data, then we have a *population*. The average weight of all ready-to-harvest pineapples in the field is a *parameter*.

- (b) Suppose the researchers also want data on taste. A panel of tasters rates the pineapples according to the categories “poor,” “acceptable,” and “good.” Only some of the pineapples are included in the taste test. In this case, the *variable* is taste. This is a *qualitative* or *categorical* variable. Because only some of the pineapples in the field are included in the study, we have a *sample*. The proportion of pineapples in the sample with a taste rating of “good” is a *statistic*.

Throughout this text, you will encounter *guided exercises* embedded in the reading material. These exercises are included to give you an opportunity to work immediately with new ideas. The questions guide you through appropriate analysis. Cover the answers on the right side (an index card will fit this purpose). After you have thought about or written down *your own response*, check the answers. If there are several parts to an exercise, check each part before you continue. You should be able to answer most of these exercise questions, but don’t skip them—they are important.

## GUIDED EXERCISE 1

### USING BASIC TERMINOLOGY

Television station QUE wants to know the proportion of TV owners in Virginia who watch the station’s new program at least once a week. The station asks a group of 1000 TV owners in Virginia if they watch the program at least once a week.

- |   |   |  |
|---|---|--|
| (a) Identify the individuals of the study and the variable.   | ➔ | The individuals are the 1000 TV owners surveyed. The variable is the response does, or does not, watch the new program at least once a week. |
| (b) Do the data comprise a sample? If so, what is the underlying population?  | ➔ | The data comprise a sample of the population of responses from all TV owners in Virginia.  |
| (c) Is the variable qualitative or quantitative?  | ➔ | Qualitative—the categories are the two possible responses, does or does not watch the program.   |
| (d) Identify a quantitative variable that might be of interest.   | ➔ | Age or income might be of interest.  |
| (e) Is the proportion of viewers in the sample who watch the new program at least once a week a statistic or a parameter? | ➔ | Statistic—the proportion is computed from sample data.   |

## LEVELS OF MEASUREMENT: NOMINAL, ORDINAL, INTERVAL, RATIO

We have categorized data as either qualitative or quantitative. Another way to classify data is according to one of the four *levels of measurement*. These levels indicate the type of arithmetic that is appropriate for the data, such as ordering, taking differences, or taking ratios.

### Levels of Measurement

#### Nominal level

#### Ordinal level

#### Interval level

#### Ratio level

### LEVELS OF MEASUREMENT

The **nominal level of measurement** applies to data that consist of names, labels, or categories. There are no implied criteria by which the data can be ordered from smallest to largest.

The **ordinal level of measurement** applies to data that can be arranged in order. However, differences between data values either cannot be determined or are meaningless.

The **interval level of measurement** applies to data that can be arranged in order. In addition, differences between data values are meaningful.

The **ratio level of measurement** applies to data that can be arranged in order. In addition, both differences between data values and ratios of data values are meaningful. Data at the ratio level have a true zero.

## EXAMPLE 2

### LEVELS OF MEASUREMENT

Identify the type of data.

- (a) Taos, Acoma, Zuni, and Cochiti are the names of four Native American pueblos from the population of names of all Native American pueblos in Arizona and New Mexico.

**SOLUTION:** These data are at the *nominal* level. Notice that these data values are simply names. By looking at the name alone, we cannot determine if one name is “greater than or less than” another. Any ordering of the names would be numerically meaningless.

- (b) In a high school graduating class of 319 students, Jim ranked 25th, June ranked 19th, Walter ranked 10th, and Julia ranked 4th, where 1 is the highest rank.

**SOLUTION:** These data are at the *ordinal* level. Ordering the data clearly makes sense. Walter ranked higher than June. Jim had the lowest rank, and Julia the highest. However, numerical differences in ranks do not have meaning. The difference between June’s and Jim’s ranks is 6, and this is the same difference that exists between Walter’s and Julia’s ranks. However, this difference doesn’t really mean anything significant. For instance, if you looked at grade point average, Walter and Julia may have had a large gap between their grade point averages, whereas June and Jim may have had closer grade point averages. In any ranking system, it is only the relative standing that matters. Differences between ranks are meaningless.

- (c) Body temperatures (in degrees Celsius) of trout in the Yellowstone River.

**SOLUTION:** These data are at the *interval* level. We can certainly order the data, and we can compute meaningful differences. However, for Celsius-scale temperatures, there is not an inherent starting point. The value  $0^{\circ}\text{C}$  may seem to be a starting point, but this value does not indicate the state of “no heat.” Furthermore, it is not correct to say that  $20^{\circ}\text{C}$  is twice as hot as  $10^{\circ}\text{C}$ .



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Korban Schwab/Stockphoto.com

(d) Length of trout swimming in the Yellowstone River.

**SOLUTION:** These data are at the *ratio* level. An 18-inch trout is three times as long as a 6-inch trout. Observe that we can divide 6 into 18 to determine a meaningful *ratio* of trout lengths.

In summary, there are four levels of measurement. The nominal level is considered the lowest, and in ascending order we have the ordinal, interval, and ratio levels. In general, calculations based on a particular level of measurement may not be appropriate for a lower level.

## PROCEDURE

### HOW TO DETERMINE THE LEVEL OF MEASUREMENT

The levels of measurement, listed from lowest to highest, are nominal, ordinal, interval, and ratio. To determine the level of measurement of data, state the *highest level* that can be justified for the entire collection of data. Consider which calculations are suitable for the data.

Level of Measurement	Suitable Calculation
Nominal	We can put the data into categories.
Ordinal	We can order the data from smallest to largest or “worst” to “best.” Each data value can be <i>compared</i> with another data value.
Interval	We can order the data and also take the differences between data values. At this level, it makes sense to compare the differences between data values. For instance, we can say that one data value is 5 more than or 12 less than another data value.
Ratio	We can order the data, take differences, and also find the ratio between data values. For instance, it makes sense to say that one data value is twice as large as another.

## WHAT DOES THE LEVEL OF MEASUREMENT TELL US?

The level of measurement tells us which arithmetic processes are appropriate for the data. This is important because different statistical processes require various kinds of arithmetic. In some instances all we need to do is count the number of data that meet specified criteria. In such cases nominal (and higher) data levels are all appropriate. In other cases we need to order the data, so nominal data would not be suitable. Many other statistical processes require division, so data need to be at the ratio level. Just keep the nature of the data in mind before beginning statistical computations.

## GUIDED EXERCISE 2

### LEVELS OF MEASUREMENT

The following describe different data associated with a state senator. For each data entry, indicate the corresponding *level of measurement*.

(a) The senator’s name is Sam Wilson.

➔ Nominal level

(b) The senator is 58 years old.

➔ Ratio level. Notice that age has a meaningful zero. It makes sense to give age ratios. For instance, Sam is twice as old as someone who is 29.

*Continued*

GUIDED EXERCISE 2 *continued*

- |   |   |  |
|---|---|--|
| (c) The years in which the senator was elected to the Senate are 2000, 2006, and 2012.  | ➔ | Interval level. Dates can be ordered, and the difference between dates has meaning. For instance, 2006 is 6 years later than 2000. However, ratios do not make sense. The year 2000 is not twice as large as the year 1000. In addition, the year 0 does not mean “no time.” |
| (d) The senator’s total taxable income last year was \$878,314.   | ➔ | Ratio level. It makes sense to say that the senator’s income is 10 times that of someone earning \$87,831.40.  |
| (e) The senator surveyed his constituents regarding his proposed water protection bill. The choices for response were strong support, support, neutral, against, or strongly against. | ➔ | Ordinal level. The choices can be ordered, but there is no meaningful numerical difference between two choices.  |
| (f) The senator’s marital status is “married.”  | ➔ | Nominal level  |
| (g) A leading news magazine claims the senator is ranked seventh for his voting record on bills regarding public education.   | ➔ | Ordinal level. Ranks can be ordered, but differences between ranks may vary in meaning.  |

**CRITICAL THINKING**

“Data! Data! Data!” he cried impatiently. “I can’t make bricks without clay.” Sherlock Holmes said these words in *The Adventure of the Copper Beeches* by Sir Arthur Conan Doyle.

Reliable statistical conclusions require reliable data. This section has provided some of the vocabulary used in discussing data. As you read a statistical study or conduct one, pay attention to the nature of the data and the ways they were collected.

When you select a variable to measure, be sure to specify the process and requirements for measurement. For example, if the variable is the weight of ready-to-harvest pineapples, specify the unit of weight, the accuracy of measurement, and maybe even the particular scale to be used. If some weights are in ounces and others in grams, the data are fairly useless.

Another concern is whether or not your measurement instrument truly measures the variable. Just asking people if they know the geographic location of the island nation of Fiji may not provide accurate results. The answers may reflect the fact that the respondents want you to think they are knowledgeable. Asking people to locate Fiji on a map may give more reliable results.

The level of measurement is also an issue. You can put numbers into a calculator or computer and do all kinds of arithmetic. However, you need to judge whether the operations are meaningful. For ordinal data such as restaurant rankings, you can’t conclude that a 4-star restaurant is “twice as good” as a 2-star restaurant, even though the number 4 is twice 2.

*Continued*