Engineering Mechanics: Statics and Dynamics

Third Edition

McGraw Hill

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Robert J. Witt | Michael E. Plesha

Engineering Mechanics

STATICS & DYNAMICS

THIRD EDITION

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The authors thank their families for their patience, understanding, and, most importantly, encouragement during the long years it took to bring these books to completion. Without their support, none of this would have been possible.

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PREFACE

Welcome to statics and dynamics! With this book we hope to provide a teaching and learning experience that is not only effective but also motivates your study and application of statics and dynamics.

The major objectives of this book are to help you

1. Learn the fundamental principles of statics and dynamics; and
2. Gain the skills needed to apply these principles in the modeling of real-life problems and for carrying out engineering design.

To achieve the objectives cited above, we have developed this book as follows. First, we provide a rigorous introduction to the fundamental principles. In a constantly changing technological landscape, it is by understanding fundamentals that you will be able to adapt to new technologies. This book places great emphasis on developing problem-solving skills. The need for an engineer to be able to accurately model real-life problems and solve them is obvious. Beyond this, it is only by mastering problem-solving skills that a true, deep understanding of fundamentals can be achieved. Second, we incorporate pedagogical principles that research in math, science, and engineering education has identified as essential for improving learning. These include teaching concepts in parallel with problem-solving skills, identifying and addressing misconceptions, and assessing conceptual understanding via specific problems. Third, we have made *modeling* the underlying theme of our approach to problem solving. We believe that modeling, understood as the making of reasonable assumptions to reduce a real-life problem to a simpler but tractable problem, is also something that must be taught and discussed alongside the basic principles. Fourth, we emphasize a systematic approach to solving every problem, an integral part of which is creating the aforementioned model. We believe these features make this book new and unique, and we hope that they will help you learn and master statics and dynamics.

Several design problems are presented where appropriate throughout the book. These problems can be tackled with the knowledge and skills that are typical of statics and dynamics, although the use of mathematical software is sometimes helpful. These problems are open ended, they allow you to show creativity in developing solutions that solve important and realistic engineering problems, and their solution requires the definition of a parameter space in which the statics and dynamics of the system must be analyzed. Design topics include methods of design, parametric analysis, issues of professional responsibility, ethics, communication, and more.

After studying statics and dynamics, you should have a thorough understanding of the fundamental principles, and, at a minimum, key points should remain in your memory for the rest of your life. We say this with a full appreciation that some of you will have careers with new and unexpected directions. Regardless of your eventual professional responsibilities, knowledge of the fundamentals of statics and dynamics will help you to be technically literate. If you *are* actively engaged in the practice of engineering and/or the sciences, then your needs go well beyond mere technical literacy, and you must also be accomplished at applying these fundamentals so that you can study more advanced subjects that build on statics and dynamics, and because you will apply concepts of statics and dynamics on a daily basis in your career.

Why Another Statics and Dynamics Series?

These books provide thorough coverage of all the pertinent topics traditionally associated with statics and dynamics. Indeed, many of the currently available texts also provide this. However, these texts offer several major innovations that enhance the learning objectives and outcomes in these subjects.

What Then Are the Major Differences Between These Books and Other Engineering Mechanics Texts?

A Consistent and Systematic Approach to Problem Solving One of the main objectives of these texts is to foster the habit of solving problems using a systematic approach. Therefore, the example problems in these texts follow a structured problem-solving methodology that will help you develop your problem-solving skills not only in statics and dynamics, but also in all other mechanics subjects that follow. This structured problem-solving approach consists of the following steps: Road Map, Modeling, Governing Equations, Computation, and Discussion & Verification. The Road Map provides some of the general objectives of the problem and develops a strategy for how the solution will be developed. Modeling is next, where a real-life problem is idealized by a model. This step results in the creation of a free body diagram and the selection of the balance laws needed to solve the problem. In dynamics, it is difficult to separate these steps, so they are usually done in combination. The Governing Equations step is devoted to writing all the equations needed to solve the problem. These equations typically include the equilibrium equations or balance laws and, depending upon the particular problem, force laws (e.g., spring law, failure criteria, frictional sliding criteria and/or friction laws) and kinematic equations. In the Computation step, the governing equations are solved. In the final step, Discussion & Verification, the solution is interrogated to ensure that it is meaningful and accurate. This problem-solving methodology is followed for all examples that involve equilibrium concepts or balance principles. Some problems (e.g., determination of the center of mass for an object or kinematic problems) do not involve equilibrium concepts, and for these the Modeling step is not needed.

Contemporary Examples, Problems, and Applications The examples, homework problems, and design problems were carefully constructed to help show you how the various topics of statics and dynamics are used in engineering practice. Statics and dynamics are immensely important subjects in modern engineering and science, and one of our goals is to excite you about these subjects and the career that lies ahead of you.

A Focus on Design A major difference between these texts and other books is the systematic incorporation of design and modeling of real-life problems throughout. In statics, topics include important discussions on design, ethics, and professional responsibility. In Dynamics, the emphasis is on parametric analysis and motion over ranges of time and space. These books show you that meaningful engineering design is possible using the concepts of statics and dynamics. Not only is the ability to develop a design very satisfying, but it also helps you develop a greater understanding of basic concepts and helps sharpen your ability to apply these concepts. Because the main focus of statics and dynamics textbooks should be the establishment of a firm understanding of basic concepts and correct problem-solving techniques, design topics do not have an overbearing presence in the books. Rather, design topics are included where they are most appropriate. While some of the discussions on design could be described as "common sense," such a characterization trivializes the importance and necessity for discussing pertinent issues such as safety, uncertainty in determining loads, the designer's responsibility to anticipate uses, even unintended uses, communications, ethics, and uncertainty in workmanship. Perhaps the most important feature of our inclusion of design and modeling topics is that you get a glimpse of what engineering is about and where your career in engineering is headed. The book is structured so that design topics and design problems are offered in a variety of places, and it is possible to pick when and where the coverage of design is most effective.

Computational Tools Some examples and problems are appropriate for solution using computer software. The use of computers extends the types of problems that can be solved while alleviating the burden of solving equations. Such examples and problems give you insight into the power of computer tools and further insight into how statics and dynamics are used in engineering practice.

Modern Pedagogy Numerous modern pedagogical elements have been included. These elements are designed to reinforce concepts, and they provide additional information to help you make meaningful connections with real-world applications. Marginal notes (i.e., Helpful Information, Common Pitfalls, Interesting Facts, and Concept Alerts) help you place topics, ideas, and examples in a larger context. These notes will help you study (e.g., Helpful Information and Common Pitfalls), will provide real-world examples of how different aspects of statics and dynamics are used (e.g., Interesting Facts), and will drive home important concepts or help dispel misconceptions (e.g., Concept Alerts and Common Pitfalls). Mini-Examples are used throughout the text to immediately and quickly illustrate a point or concept without making readers wait for the worked-out examples at the end of the section.

Answers to Problems The answers to most even-numbered problems have been included in the back matter for ease of use as Appendix B. Providing answers in this manner allows for the inclusion of more complex information than would otherwise be possible. In addition to final numerical and/or symbolic answers, shear and moment diagrams and/or plots for Computer Problems are included.

Changes to the Third Edition

The third editions of *Engineering Mechanics: Statics* and *Engineering Mechanics: Dynamics* retain all of the major pedagogical features of the previous editions, including a structured problem-solving methodology for all example problems, contemporary engineering applications in the example problems and homework exercises, the inclusion of engineering design and its implications for problem solving and applications, and use of computational tools where applicable. In addition, as a result of the author-based typesetting process, the outstanding accuracy of the earlier editions has been preserved, leading to books whose accuracy is unrivaled among textbooks.

The third editions contain revised and enhanced textual discussions and example problems, additional figures where effective, and new homework exercises. In Connect, the online homework system, there are significant updates, including an auto-graded FBD tool and interactive learning tools. These interactive assignments help reinforce what is being covered in the text and show students how to tie the material to real-world situations. These tools complement the hundreds of auto-graded, algorithmic-exercises that are included in Connect from the text.

The following individuals have been instrumental in ensuring the highest standard of content and accuracy. We are deeply indebted to them for their tireless efforts.

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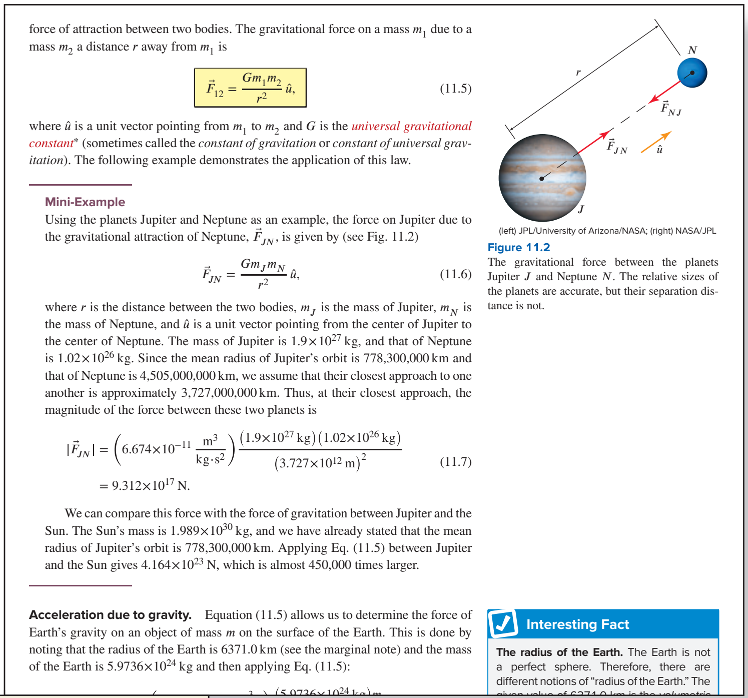
Mini-Examples

Mini-examples are used throughout the text to immediately and quickly illustrate a point or concept without having to wait for the worked-out examples at the end of the section.



force of attraction between two bodies. The gravitational force on a mass m_1 due to a mass m_2 a distance r away from m_1 is

$$\boxed{\vec{F}_{12} = \frac{Gm_1 m_2}{r^2}\,\hat{u},} \tag{11.5}$$

where $\hat{u}$ is a unit vector pointing from m_1 to m_2 and G is the *universal gravitational constant** (sometimes called the *constant of gravitation* or *constant of universal gravitation*). The following example demonstrates the application of this law.

Mini-Example

Using the planets Jupiter and Neptune as an example, the force on Jupiter due to the gravitational attraction of Neptune, $\vec{F}_{JN}$, is given by (see Fig. 11.2)

$$\vec{F}_{JN} = \frac{Gm_J m_N}{r^2}\,\hat{u}, \tag{11.6}$$

where r is the distance between the two bodies, m_J is the mass of Jupiter, m_N is the mass of Neptune, and $\hat{u}$ is a unit vector pointing from the center of Jupiter to the center of Neptune. The mass of Jupiter is 1.9×10^{27} kg, and that of Neptune is 1.02×10^{26} kg. Since the mean radius of Jupiter's orbit is 778,300,000 km and that of Neptune is 4,505,000,000 km, we assume that their closest approach to one another is approximately 3,727,000,000 km. Thus, at their closest approach, the magnitude of the force between these two planets is

$$|\vec{F}_{JN}| = \left(6.674\times10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\right)\frac{(1.9\times10^{27}\,\text{kg})(1.02\times10^{26}\,\text{kg})}{(3.727\times10^{12}\,\text{m})^2} \tag{11.7}$$
$$= 9.312\times10^{17}\,\text{N}.$$

We can compare this force with the force of gravitation between Jupiter and the Sun. The Sun's mass is 1.989×10^{30} kg, and we have already stated that the mean radius of Jupiter's orbit is 778,300,000 km. Applying Eq. (11.5) between Jupiter and the Sun gives 4.164×10^{23} N, which is almost 450,000 times larger.

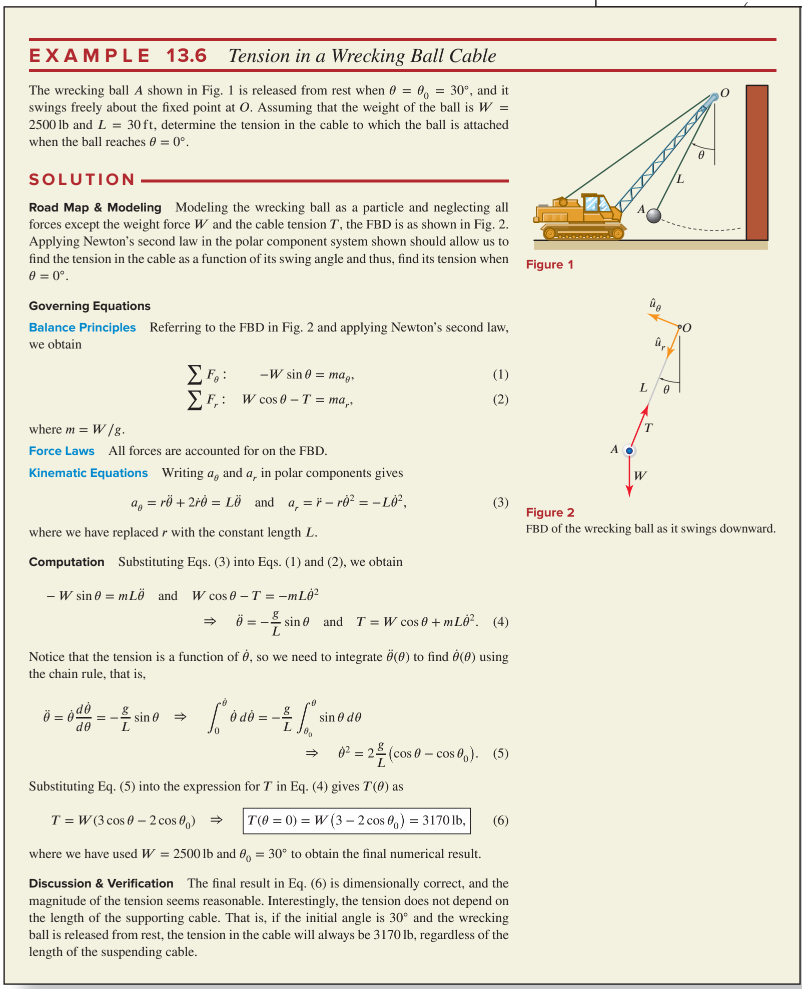
Acceleration due to gravity. Equation (11.5) allows us to determine the force of Earth's gravity on an object of mass m on the surface of the Earth. This is done by noting that the radius of the Earth is 6371.0 km (see the marginal note) and the mass of the Earth is 5.9736×10^{24} kg and then applying Eq. (11.5):

(left) JPL/University of Arizona/NASA; (right) NASA/JPL

Figure 11.2
The gravitational force between the planets Jupiter J and Neptune N. The relative sizes of the planets are accurate, but their separation distance is not.

Interesting Fact

The radius of the Earth. The Earth is not a perfect sphere. Therefore, there are different notions of "radius of the Earth." The



EXAMPLE 13.6 *Tension in a Wrecking Ball Cable*

The wrecking ball A shown in Fig. 1 is released from rest when $\theta = \theta_0 = 30°$, and it swings freely about the fixed point at O. Assuming that the weight of the ball is $W = 2500$ lb and $L = 30$ ft, determine the tension in the cable to which the ball is attached when the ball reaches $\theta = 0°$.

SOLUTION

Road Map & Modeling Modeling the wrecking ball as a particle and neglecting all forces except the weight force W and the cable tension T, the FBD is as shown in Fig. 2. Applying Newton's second law in the polar component system shown should allow us to find the tension in the cable as a function of its swing angle and thus, find its tension when $\theta = 0°$.

Governing Equations

Balance Principles Referring to the FBD in Fig. 2 and applying Newton's second law, we obtain

$$\sum F_\theta: \quad -W\sin\theta = ma_\theta, \tag{1}$$
$$\sum F_r: \quad W\cos\theta - T = ma_r, \tag{2}$$

where $m = W/g$.

Force Laws All forces are accounted for on the FBD.

Kinematic Equations Writing a_θ and a_r in polar components gives

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = L\ddot{\theta} \quad\text{and}\quad a_r = \ddot{r} - r\dot{\theta}^2 = -L\dot{\theta}^2, \tag{3}$$

where we have replaced r with the constant length L.

Computation Substituting Eqs. (3) into Eqs. (1) and (2), we obtain

$$-W\sin\theta = mL\ddot{\theta} \quad\text{and}\quad W\cos\theta - T = -mL\dot{\theta}^2$$
$$\Rightarrow\quad \ddot{\theta} = -\frac{g}{L}\sin\theta \quad\text{and}\quad T = W\cos\theta + mL\dot{\theta}^2. \tag{4}$$

Notice that the tension is a function of $\dot{\theta}$, so we need to integrate $\ddot{\theta}(\theta)$ to find $\dot{\theta}(\theta)$ using the chain rule, that is,

$$\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta} = -\frac{g}{L}\sin\theta \quad\Rightarrow\quad \int_0^{\dot{\theta}} \dot{\theta}\,d\dot{\theta} = -\frac{g}{L}\int_{\theta_0}^{\theta}\sin\theta\,d\theta$$
$$\Rightarrow\quad \dot{\theta}^2 = 2\frac{g}{L}(\cos\theta - \cos\theta_0). \tag{5}$$

Substituting Eq. (5) into the expression for T in Eq. (4) gives $T(\theta)$ as

$$T = W(3\cos\theta - 2\cos\theta_0) \quad\Rightarrow\quad \boxed{T(\theta = 0) = W(3 - 2\cos\theta_0) = 3170\,\text{lb},} \tag{6}$$

where we have used $W = 2500$ lb and $\theta_0 = 30°$ to obtain the final numerical result.

Discussion & Verification The final result in Eq. (6) is dimensionally correct, and the magnitude of the tension seems reasonable. Interestingly, the tension does not depend on the length of the supporting cable. That is, if the initial angle is 30° and the wrecking ball is released from rest, the tension in the cable will always be 3170 lb, regardless of the length of the suspending cable.

Figure 1

Figure 2
FBD of the wrecking ball as it swings downward.

Examples

Consistent Problem-Solving Methodology

Every problem in the text employs a carefully defined problem-solving methodology to encourage systematic problem formulation while reinforcing the steps needed to arrive at correct and realistic solutions.

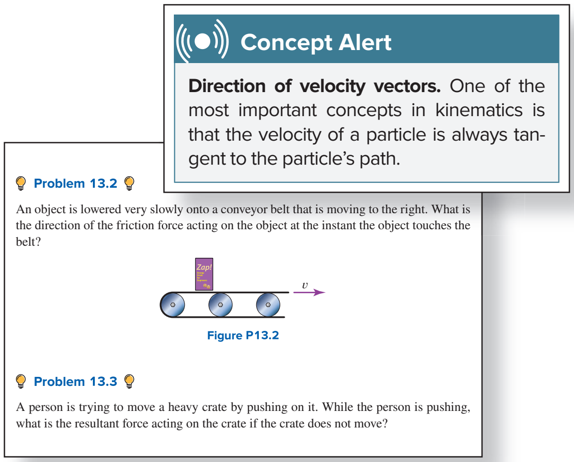
Each example problem contains these steps:

- **Road Map**
- **Modeling**
- **Governing Equations**
- **Computation**
- **Discussion & Verification**

Some examples include a Closer Look (noted with a magnifying glass icon) that offers additional insight into the example.

Concept Alerts and Concept Problems

Two additional features are the Concept Alerts and the Concept Problems. These have been included because research has shown (and it has been our experience) that even though you may do quite well in a science or engineering course, your conceptual understanding may be lacking. **Concept Alerts** are marginal notes and are used to drive home important concepts (or help dispel misconceptions) that are related to the material being developed at that point in the text. **Concept Problems** are mixed in with the problems that appear at the end of each section. These are questions designed to get you thinking about the application of a concept or idea presented within that section. They should never require calculation and should require answers of no more than a few sentences.



Concept Alert

Direction of velocity vectors. One of the most important concepts in kinematics is that the velocity of a particle is always tangent to the particle's path.

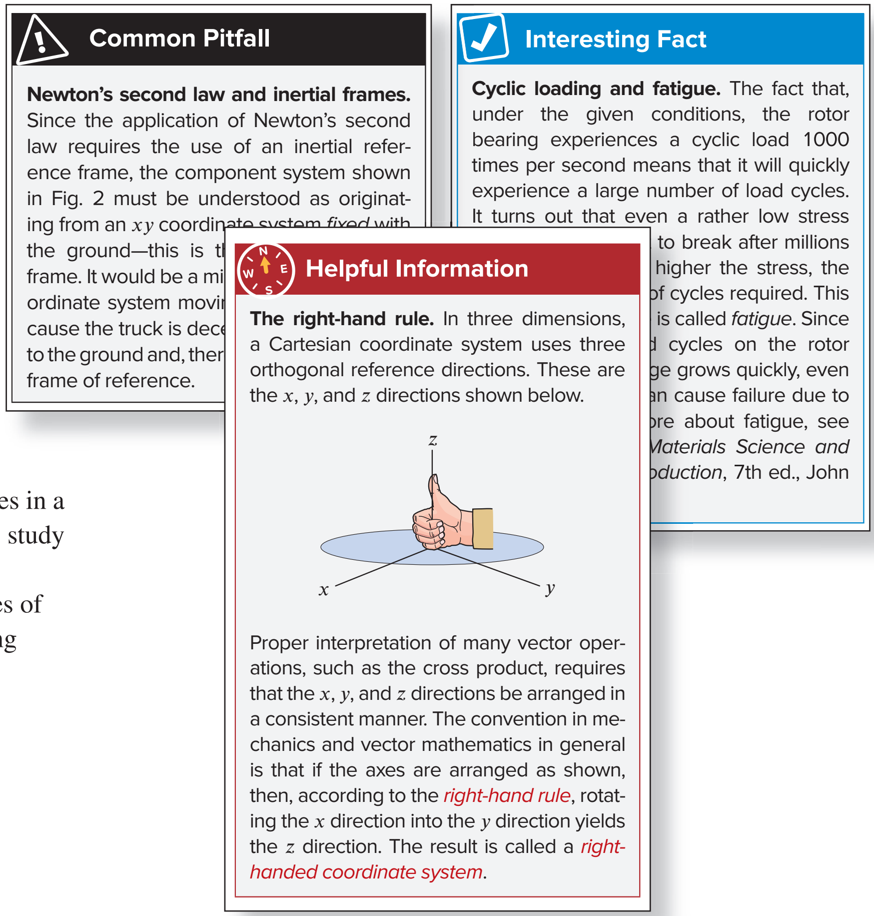
Problem 13.2

An object is lowered very slowly onto a conveyor belt that is moving to the right. What is the direction of the friction force acting on the object at the instant the object touches the belt?

Figure P13.2

Problem 13.3

A person is trying to move a heavy crate by pushing on it. While the person is pushing, what is the resultant force acting on the crate if the crate does not move?



Common Pitfall

Newton's second law and inertial frames. Since the application of Newton's second law requires the use of an inertial reference frame, the component system shown in Fig. 2 must be understood as originating from an xy coordinate system *fixed* with the ground—this is t frame. It would be a mi ordinate system movi cause the truck is dec to the ground and, the frame of reference.

Interesting Fact

Cyclic loading and fatigue. The fact that, under the given conditions, the rotor bearing experiences a cyclic load 1000 times per second means that it will quickly experience a large number of load cycles. It turns out that even a rather low stress to break after millions higher the stress, the of cycles required. This is called *fatigue*. Since d cycles on the rotor ge grows quickly, even an cause failure due to ore about fatigue, see *Materials Science and oduction*, 7th ed., John

Helpful Information

The right-hand rule. In three dimensions, a Cartesian coordinate system uses three orthogonal reference directions. These are the x, y, and z directions shown below.

Proper interpretation of many vector operations, such as the cross product, requires that the x, y, and z directions be arranged in a consistent manner. The convention in mechanics and vector mathematics in general is that if the axes are arranged as shown, then, according to the *right-hand rule*, rotating the x direction into the y direction yields the z direction. The result is called a *right-handed coordinate system*.

Marginal Notes

Marginal notes have been implemented that will help place topics, ideas, and examples in a larger context. This feature will help students study (using **Helpful Information** and **Common Pitfalls**) and will provide real-world examples of how different aspects of statics are used (using **Interesting Facts**).

Sections and End of Section Summary

Each chapter is organized into several sections. There is a wealth of information and features within each section, including examples, problems, marginal notes, and other pedagogical aids. Each section concludes with an end of section summary that succinctly summarizes that section. In many cases, cross-referenced important equations are presented again for review and reinforcement before the student proceeds to the examples and homework problems.

19.2 Undamped Forced Vibration

Many systems are *forced* to vibrate by an external excitation. This section is devoted to the forced vibration of mechanical systems.

Standard form of the forced harmonic oscillator

A standard forced harmonic oscillator is shown in Fig. 19.11, in which the block of mass m is attached to a fixed support by a linear spring of constant k and is also being driven by the time-dependent force $P(t) = F_0 \sin \omega_0 t$. Modeling the block as a particle, its FBD is as shown in Fig. 19.12, where F_s is the spring force acting on the block. Summing forces in the x direction, we obtain

$$\sum F_x: \quad P(t) - F_s = ma_x, \tag{19.29}$$

where the force law is given by $F_s = kx$ and the kinematic equation is $a_x = \ddot{x}$. Substituting these relations, as well as $P(t)$ into Eq. (19.29), we obtain

$$F_0 \sin \omega_0 t - kx = m\ddot{x} \quad \Rightarrow \quad \ddot{x} + \frac{k}{m}x = \frac{F_0}{m} \sin \omega_0 t. \tag{19.30}$$

Noting that $\omega_n^2 = k/m$, this last equation becomes

$$\boxed{\ddot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega_0 t,} \tag{19.31}$$

which is the *standard form of the forced harmonic oscillator equation*. It is a *nonhomogeneous* version of Eq. (19.12) on p. 1291 as a result of the term $(F_0/m)\sin \omega_0 t$. The term on the right-hand side of Eq. (19.31) is a function of *only* the independent variable t. It is often called a *forcing function* because it forces the system to vibrate. This particular type of forcing is harmonic because it is a harmonic function of time.

The theory of differential equations tells us that the *general solution* of Eq. (19.31) is the sum of the *complementary solution* $x_c(t)$ and a *particular solution* $x_p(t)$. The *complementary solution** is the solution of the associated homogeneous equation

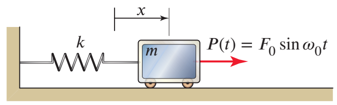


Figure 19.11
A forced harmonic oscillator whose equation of motion is given by Eq. (19.31) with $\omega_n = \sqrt{k/m}$. The position x is measured from the equilibrium position of the system when $F_0 = 0$.

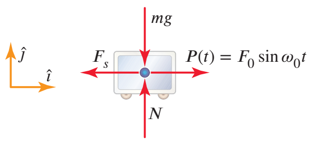


Figure 19.12
FBD of the forced harmonic oscillator in Fig. 19.11.

Interesting Fact

How practical is harmonic forcing? The answer to this question lies in an amazing result due to Jean Baptiste Joseph Fourier ... contributors. It ...iecewise smooth ...ted by an infinite ...es (called *Fourier* ...ier). *This means* ...*can be regarded* ...*functions!* In ad... ...f the left side of ...), it turns out that ...tion for a sum of ...simply the sum of ...r each individual ...ese results taken ... to easily obtain ...with *any* periodic ... forced harmonic ...se periodic forc... ...neering systems, ...portant results in

End of Section Summary

When a harmonic oscillator is subject to harmonic forcing, the standard form of the equation of motion is

Eq. (19.31), p. 1307

$$\ddot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega_0 t,$$

where F_0 is the amplitude of the forcing and ω_0 is its frequency (see Fig. 19.16). The general solution to this equation consists of the sum of the complementary solution and a particular solution. The *complementary solution* x_c is the solution of the associated homogeneous equation, which is given by, for example, Eq. (19.13). For $\omega_0 \neq \omega_n$, a particular solution was found to be

Eq. (19.35), p. 1308

$$x_p = \frac{F_0/k}{1 - (\omega_0/\omega_n)^2} \sin \omega_0 t,$$

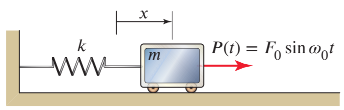


Figure 19.16
A forced harmonic oscillator whose equation of motion is given by Eq. (19.31) with $\omega_n = \sqrt{k/m}$. The position x is measured from the equilibrium position of the block.

Problems

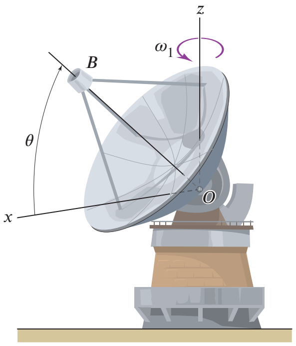


Figure P20.1 and P20.2

Problems 20.1 and 20.2

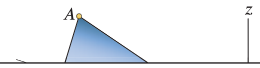
The radar dish can rotate about the vertical z axis at rate ω_1 and about the horizontal y axis (not shown in the figure) at rate $\dot{\theta}$. The distance between the center of rotation at O and the subreflector at B is ℓ.

Problem 20.1 If ω_1 and $\dot{\theta}$ are both constant, determine the velocity and acceleration of the subreflector B in terms of the elevation angle θ.

Problem 20.2 If $\omega_1(t)$ and $\dot{\theta}(t)$ are known functions of time, determine the velocity and acceleration of the subreflector B in terms of the elevation angle θ.

Problems 20.3 and 20.4

The truncated cone rolls without slipping on the xy plane. At the instant shown, the angular speed about the z axis is ω_1, and it is changing at $\dot{\omega}_1$.



Problems 12.178 and 12.179

A micro spiral pump* consists of a spiral channel attached to a stationary plate. This plate has two ports, one for fluid inlet and another for outlet, the outlet being farther from the center of the plate than the inlet. The system is capped by a rotating disk. The fluid trapped between the rotating disk and the stationary plate is put in motion by the rotation of the top disk, which pulls the fluid through the spiral channel.

Problem 12.178 Consider a spiral channel with the geometry given by the equation $r = \eta\theta + r_0$, where $r_0 = 146\,\mu$m is the starting radius, r is the distance from the spin axis, and θ, measured in radians, is the angular position of a point in the spiral channel. Assume that the radius at the outlet is $r_{out} = 190\,\mu$m, that the top disk rotates with a constant angular speed ω, and that the fluid particles in contact with the rotating disk are essentially stuck to it. Determine the constant η and the value of ω (in rpm) such that after 1.25 rev of the top disk, the speed of the particles in contact with this disk is $v = 0.5$ m/s at the outlet.

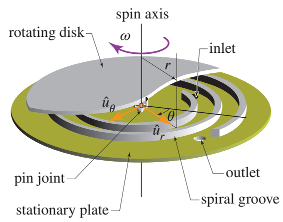


Figure P12.178 and P12.179

Modern Problems

Problems of varying difficulty follow each section. These problems allow students to develop their ability to apply concepts of statics and dynamics on their own. The most common question asked by students is "How do I set this problem up?" What is really meant by this question is "How do I develop a good mathematical model for this problem?" The only way to develop this ability is by practicing numerous problems. Answers to most even-numbered problems appear in Appendix B. Providing answers in this manner allows for more complex information than would otherwise be possible.

Furthermore, the free body diagrams that are provided for some problems as part of the model for that problem will give students ample opportunity to practice constructing FBDs on their own for extra problems. Appendix B gives examples of the additional information provided for particular problems. Each problem in the book is accompanied by a thermometer icon that indicates the approximate level of difficulty. Those considered to be "introductory" are indicated with the symbol . Problems considered to be "representative" are indicated with the symbol , and problems that are considered to be "challenging" are indicated with the symbol .

Engineering Design and Design Problems

Several design problems are presented where appropriate throughout the book. These problems can be tackled with the knowledge and skill set that are typical of introductory-level courses, although the use of mathematical software is strongly recommended. These problems are open ended, and their solution requires the definition of a parameter space in which the statics or dynamics of the system must be analyzed. In this textbook we have chosen to emphasize the role played by *parametric analyses* in the overall design process, as opposed to cost-benefit analyses or the choice of specific materials and/or components.

Design Problems

Design Problem 17.1

Revisit the calculations done at the beginning of the chapter concerning the determination of the maximum acceleration that can be achieved by a motorcycle without causing the front wheel to lift off the ground. Specifically, construct a new model of the motorcycle by selecting a real-life motorcycle and researching its geometry and inertia properties, including the inertia properties of the wheels. Then analyze your model to determine how the maximum acceleration in question depends on the horizontal and vertical positions of the center of mass with respect to the points of contact between the ground and the wheels. Include in your analysis a comparison of results that account for the inertia of the front wheel with results that neglect the inertia of the front wheel.

Guitar Studio/Shutterstock

Figure DP17.1

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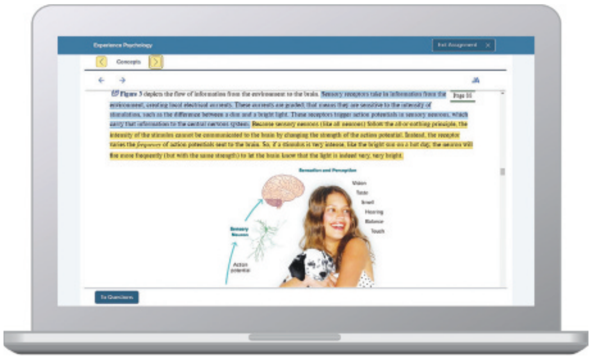

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1 Introduction to Statics

In *statics* we study the equilibrium of bodies under the action of forces that are applied to them. Our goal is to provide an introduction to the science, skill, and art involved in modeling and designing real life mechanical systems. We begin the study of statics with an overview of the relevant history of the subject. In subsequent sections and chapters, we cover those elements of physics and mathematics (especially vectors) needed to analyze the equilibrium of particles and rigid bodies. Throughout the book are discussions and applications of engineering design.

Kapook2981/iStock/Getty Images

The Infinity Bridge crossing the River Tees in England is a dual arch bridge with steel arches and concrete decking. Concepts from statics formed the basis for the analysis and design of this bridge.

1.1 Engineering and Statics

Engineers design structures, machines, processes, and much more for the benefit of humankind. In the process of doing this, an engineer must answer questions such as “Is it strong enough?” “Will it last long enough?” and “Is it safe enough?” To answer these questions requires the ability to quantify important phenomena in the design or system at hand, and to compare these measures with known criteria for what is acceptable and what is not. To do this requires an engineer to have thorough knowledge of science, mathematics, and computational tools, and the creativity to exploit the laws of nature to develop new designs. Central to all of this is the ability to idealize real life problems with mathematical models that capture the essential science of the problem, yet are tractable enough to be analyzed. Proficiency in doing this is a characteristic that sets engineering apart from the pure sciences.

In most engineering disciplines, understanding the response of materials or objects subjected to forces is important, and the fundamental science concepts

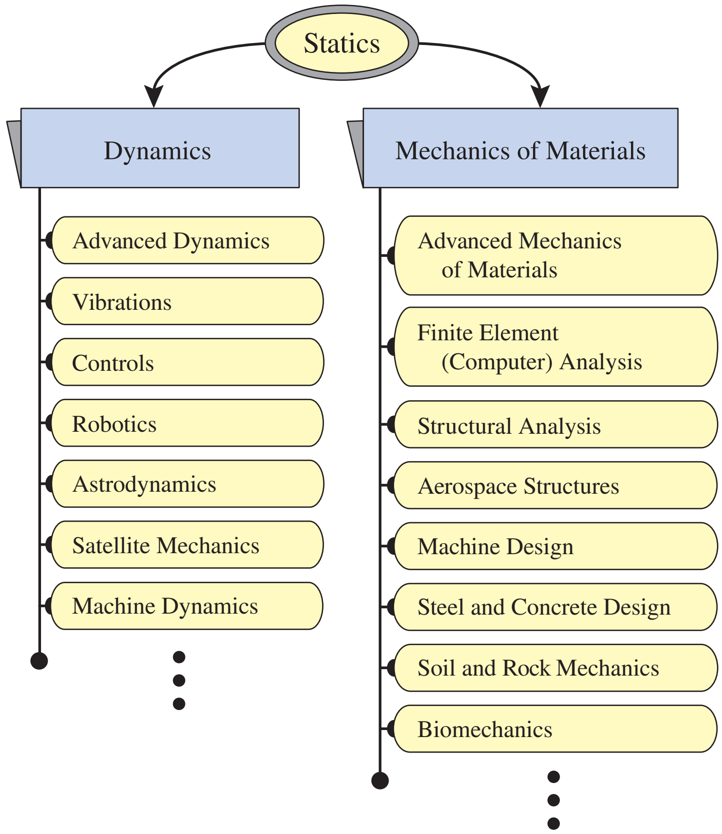

Figure 1.1. Hierarchy of subject matter and courses studied by many engineering students. Courses in statics, dynamics, and mechanics of materials provide fundamental concepts and a basis for more advanced study. Many subjects, such as vibrations and finite element analysis, draw heavily on concepts from both dynamics and mechanics of materials.

governing such response are known as *Newtonian physics*.* This book examines applications of this topic to engineering problems under the special circumstances in which a system is in force equilibrium, and this body of knowledge is called *statics*. Statics is usually the first engineering course that students take. Statics is an important subject in its own right, and it develops essential groundwork for more advanced study.

If you have read this far, then we presume you are embarking on a study of statics, using this book as an aid. Figure 1.1 shows a hierarchy of subjects, many of which you are likely to study en route to an education in engineering. Following a course in statics are introductory courses in *dynamics* and *mechanics of materials*. Dynamics studies the motion of particles and bodies subjected to forces that are not in equilibrium. Mechanics of materials introduces models for material behavior and methods for determining stresses and deformations in structures. The concepts learned in these three basic courses are used daily by almost all engineers who are concerned with the mechanical response of structures and materials!

This book will provide you with a solid and comprehensive education in statics. Often, when engineering problems are boiled down to their essential elements, they are remarkably simple to analyze. In fact, throughout most of this book, the mathe-

* When the velocity of an object is close to the speed of light, relativistic physics is required.

matics needed to analyze problems is straightforward. The bigger challenge usually lies in the idealization of a real life problem by a model, and we hope this book helps you cultivate your ability to do this.

Regarding mathematics, this book assumes you have knowledge of algebra and basic trigonometry. Later in this book, beginning in Chapter 7, basic calculus involving differentiation and integration of simple functions is used. Vectors is an important topic, and this book assumes that you have no prior knowledge of this; everything you need to know about vectors for statics will be covered in this book.

1.2 Topics That Will Be Studied in Statics

The remainder of Chapter 1 is devoted to a discussion of the physical entities and governing equations that form the basis of statics and dynamics. An important related topic is the choice of the unit system to be used. Chapter 2 introduces vectors and how they are used to represent entities such as force and position. In Chapters 3 through 6, we use statics to solve problems involving particles and systems of particles that are in equilibrium, and bodies and systems of bodies (i.e., frames and machines) that are in equilibrium. Each of these topics builds upon previous topics to enable you to model engineering problems of increasingly greater sophistication. Chapters 1 through 6 constitute the core of topics in statics.

Beginning in Chapter 7, we treat systems that have continuous distributions of properties such as mass, weight, and pressure; basic calculus is effective and is used beginning here. Chapter 8 addresses internal forces that develop within structures due to loads that are applied to them; knowledge of internal forces is essential to create designs and to address questions such as "Is it strong enough?" and "Is it safe enough?" Chapter 9 is devoted to friction, which is a type of force between contacting bodies. Friction presents some challenges to engineers to model and account for in engineering problems. Finally, Chapter 10 is devoted to moments of inertia, which characterize how area and mass are distributed; this topic is essential in dynamics and mechanics of materials, and it marks the transition from statics to these subjects.

1.3 A Brief History of Statics*

The history of statics is not a distinct subject, as it is closely intertwined with the development of dynamics and mechanics of materials. Early scientists and engineers were commonly called *philosophers*, and their noble undertaking was to use thoughtful reasoning to provide explanations for natural phenomena. Much of their focus was on understanding and describing the equilibrium of objects and the motion of celestial bodies. With few exceptions, their studies had to yield results that were intrinsically beautiful and/or compatible with the dominant religion of the time and place. What follows is a short historical survey of the major figures who profoundly influenced the development of key aspects of mechanics that are especially significant to statics.

* This history is based on the excellent works of C. Truesdell, *Essays in the History of Mechanics*, Springer-Verlag, Berlin, 1968; I. Bernard Cohen, *The Birth of a New Physics*, revised and updated edition, W. W. Norton & Company, New York, 1985; R. Dugas, *A History of Mechanics*, Dover, Mineola, NY, 1988; and James H. Williams, Jr., *Fundamentals of Applied Dynamics*, John Wiley & Sons, New York, 1996.

Interesting Fact

Early structural design codes. While most of our discussion focuses on accomplishments of philosophers, there were also significant accomplishments in the development of structural design codes over a period of thousands of years. Some of these include the ancient books of Ezekiel and Vitruvius and the secret books of the medieval masonic lodges. Additional history is given in J. Heyman, "Truesdell and the History of the Theory of Structures," a chapter in *Essays on the History of Mechanics*, edited by A. Becchi, M. Corradi, F. Foce, and O. Pedemonte, Birkhauser, Boston, 2003. These codes were largely empirical rules of proportion that provided for efficient design and construction of masonry structures. The great Greek temples, Roman aqueducts, and Gothic cathedrals are a testament to their effectiveness. While the writers of these codes were not philosophers, their engineering accomplishments were impressive.

Bruno Cossa/SOPA/Corbis

The Parthenon in Athens, Greece, was completed in 438 B.C. and is an example of early column and beam masonry construction.

For centuries, philosophers studied the equilibrium and motion of bodies with less than full understanding, and sometimes incorrect understanding. Notable early contributors include:

- Aristotle (384–322 B.C.) wrote about science, politics, economics, and biology, and he proposed what is often called a "physics of common sense." He studied levers and although he attributed their efficiency to the "magical" properties of the circle, he understood some basic concepts of the moment of a force and its effect on equilibrium. He classified objects as being either light or heavy, and he believed that light objects fall more slowly than heavy objects. He recognized that objects can move in directions other than up or down; he said that such motion is contrary to the natural motion of the body and that some force must continuously act on the body for it to move this way. Most importantly, he said that the natural state of objects is for them to be at rest.

- Archimedes (287–212 B.C.) postulated several axioms based on experimental observations of the equilibrium of levers, and using these, he proved several propositions. His work shows further understanding of the effects of the moment of a force on equilibrium. Archimedes is perhaps best known for his pioneering work on hydrostatic fluid mechanics, where one of his discoveries was that a body that floats in fluid will displace a volume of fluid whose weight is equal to that of the body. Recently, evidence has been found that he discovered some elementary concepts of calculus.

- Leonardo da Vinci (1452–1519) had bold imagination and tackled a wide variety of problems. He correctly understood the moment of a force and used the terminology *arm of the potential lever* to describe what we today call the *moment arm*. While his conclusions were wrong, he studied the equilibrium of a body supported by two strings. He also conducted experiments on the strength of structural materials.

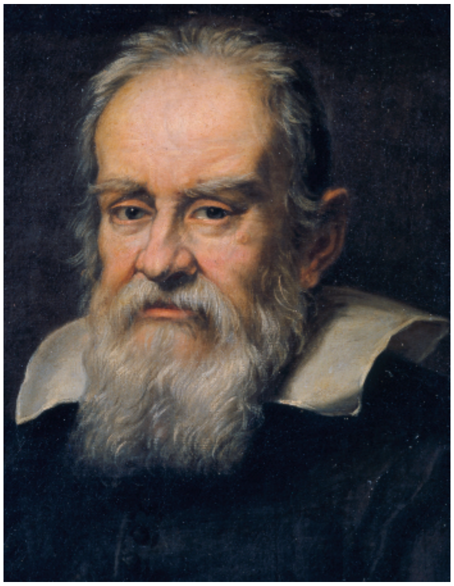

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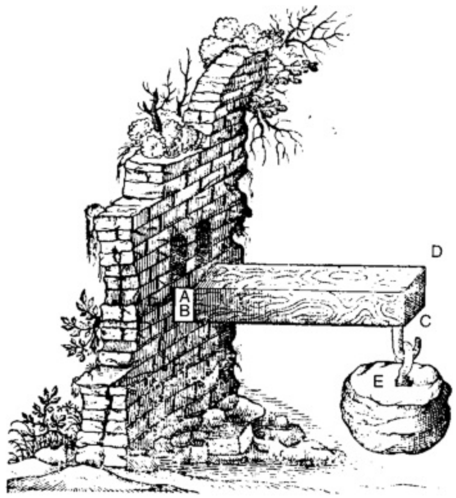
Figure 1.2
A portrait of Galileo painted in 1636 by Justus Sustermans.

Following the progress of these and many other early philosophers came the work of Galileo and Newton. With their work came rapid progress in achieving the essential elements of a theory for the motion of bodies, and their accomplishments represent the most important milestone in the history of mechanics until the work of Einstein. The contributions of Galileo and Newton are discussed in some detail in the remainder of this section.

Galileo Galilei

Galileo Galilei (1564–1642) had a strong interest in mathematics, mechanics, astronomy, heat, and magnetism. He made important contributions throughout his life, despite persecution from the church for his support of the Copernican theory that the Earth was not the center of the universe. One of his most important contributions was his thought experiment in which he concluded that a body in its natural state of motion has *constant velocity*. Galileo discovered the correct law for freely falling bodies; that is, the distance traveled by a body is proportional to the square of time. He also concluded that two bodies of different weight would fall at the same rate and that any differences are due to air resistance. Galileo developed a theory (with some minor errors) for the strength of beams, such as that shown in Fig. 1.3. He was the first to use the concept of stress as a fundamental measure of the loading a material supports, and he is viewed as the father of mechanics of materials. He also discovered that the strength of structures does not scale linearly; that is, if the dimensions of a beam are doubled, the load the beam can support does not double. He speculated that it is for this reason that trees, animals, and so on have natural limits to the size they could reach before they would fail under their own weight. More importantly, his work showed that newer, larger structures could not necessarily be built by simply scaling the dimensions of smaller structures that were successfully built.



The Picture Art Collection/Alamy Stock Photo

Figure 1.3
A sketch from Galileo's last book *Discourses on Two New Sciences*, published in 1638, where he studies the strength of beams, among several other topics.

Isaac Newton

Newton (1643[*]–1727) was one of the greatest scientists of all time. He made important contributions to optics, astronomy, mathematics, and mechanics, and his collection of three books entitled *Philosophiæ Naturalis Principia Mathematica*, or *Principia* as they are generally known, which were published in 1687, is considered by many to be the greatest collection of scientific books ever written.

In the *Principia*, Newton analyzed the motion of bodies in "resisting" and "nonresisting media." He applied his results to orbiting bodies, projectiles, pendula, and free fall near the Earth. By comparing his "law of centrifugal force" with Kepler's third law of planetary motion, Newton further demonstrated that the planets were attracted to the Sun by a force varying as the inverse square of the distance, and he generalized that all heavenly bodies mutually attract one another in the same way. In the first book of the *Principia*, Newton develops his three laws of motion; in the second book he develops some concepts in fluid mechanics, waves, and other areas of physics; and in the third book he presents his law of universal gravitation. His contributions in the first and third books are especially significant to statics and dynamics.

Newton's *Principia* was the final brick in the *foundation* of the laws that govern the motion of bodies. We say *foundation* because it took the work of Daniel Bernoulli (1700–1782), Johann Bernoulli (1667–1748), Jean le Rond d'Alembert (1717–1783), Joseph-Louis Lagrange (1736–1813), and Leonhard Euler (1707–1783) to clarify, refine, and advance the theory of dynamics into the form used today. Euler's contributions are especially notable since he used Newton's work to develop the theory for rigid body dynamics. Newton's work, along with Galileo's, also provided the foundation for the theory of mechanical behavior of deformable bodies, which is more commonly called mechanics of materials. However, it took the work of Charles-Augustin Coulomb (1736–1806), Claude Louis Marie Henri Navier (1785–1857), and Augustin Cauchy (1789–1857) to further refine the concept of stress into the form used today; the work of Robert Hooke (1635–1703) and Thomas Young (1773–1829) to develop a theory for elastic deformation of materials; and the work of Leonhard Euler (1707–1783) to consider the deformations of a structure (an elastic strip in particular).[†]

Imagno/Hulton Fine Art Collection/Getty Images

Figure 1.4
A portrait of Newton painted in 1689 by Sir Godfrey Kneller, which is owned by the 10th Earl of Portsmouth. It shows Newton before he went to London to take charge of the Royal Mint and when he was at his scientific peak.

1.4 Fundamental Principles

Space and time. Most likely you already have a good intuitive understanding of the concepts of space and time. In fact, to refine concepts of space and time is not easy and may not provide the clarification we would like. *Space* is the collection of all positions in our universe that a point may occupy. The location of a point is usually described using a coordinate system where measurements are made from some

[*] This birth date is according to the Gregorian, or "modern," calendar. According to the older Julian calendar, which was used in England at that time, Newton's birth was in 1642.

[†] Additional comments on the history of mechanics as it pertains to mechanics of materials are given in M. Vable, *Mechanics of Materials*, Oxford University Press, New York, 2002.

reference position using the coordinate system's reference directions. While selection of a reference position and directions is arbitrary, it is usually based on convenience. Because space is three-dimensional, three pieces of information, called *coordinates*, are required to locate a point in space. Most often we will use a rectangular Cartesian coordinate system where the distances to a point are measured in three orthogonal directions from a reference location. Other coordinate systems, such as spherical and cylindrical coordinates (and polar coordinates in two dimensions), are sometimes more convenient. All engineering problems are three-dimensional, but often we will be able to idealize a problem as being two-dimensional or one-dimensional. *Time* provides a measure of when an event, or sequence of events, occurs.

Mass and force. *Mass* is the amount of matter, or material, in an object. *Force* is an agency that is capable of producing motion of an object. Forces can arise from contact or interaction between objects, from gravitational attraction, from magnetic attraction, and so on. As discussed in Section 1.6, interpretation and quantification of mass and force should be viewed as being related by Newton's second law of motion. Force is discussed further in Section 1.5.

Particle. A *particle* is an object whose mass is concentrated at a point. For this reason, a particle is also called a *point mass*, and it is said to have zero volume. An important consequence of this definition is that the notion of rotational motion of a particle is meaningless. Clearly there are no true particles in nature, but under the proper circumstances it is possible to idealize real life objects as particles. Objects that are small compared to other objects and/or dimensions in a problem can often be idealized as particles. For example, to determine the orbit of a satellite around the Earth, it is probably reasonable to idealize the satellite as a particle. Objects do not necessarily need to be small to be accurately idealized as particles. For example, for the satellite orbiting Earth, the Earth is not small, but for many purposes the Earth can also be idealized as a particle.

Body and rigid body. A *body* has mass and occupies a volume of space. In nature, all bodies are deformable. That is, when a body is subjected to forces, the distances between points in the body may change. A *rigid body* is a body that is not deformable, and hence the distance between any two points in the body never changes. There are no true rigid bodies in nature, but very often we may idealize an object to be a rigid body, and this provides considerable simplification because the intricate details of how the body deforms do not need to be accounted for in an analysis. Furthermore, in statics we will be able to make precise statements about the behavior of rigid bodies, and we will establish methods of analysis that are exact.

Scalars and vectors. A *scalar* is a quantity that is completely characterized by a single number. For example, temperature, length, and density are scalars. In this book, scalars are denoted by italic symbols, such as s. A *vector* is an entity that has both size (or magnitude) and direction. Much will be said about vectors in Chapter 2, but basic notions of vectors will be useful immediately. Statements such as "my apartment is 1 mile northeast of Engineering Hall" and "I'm walking north at 3 km/h" are statements of vector quantities. In the first example, the position of one location relative to another is stated, while in the second example, the velocity is stated. In both examples, commonly used reference directions of north and east are employed. Vectors are immensely useful for describing many entities in mechanics. Vectors offer compact representation and easy manipulation, and they can be transformed. That

Concept Alert

Vectors. A *vector* is an entity that has both size and direction. Vectors are immensely useful in mechanics, and the ability to use vectors to represent force, position, and other entities is essential.

is, if a vector is known in reference to one set of coordinate directions, then using established rules for transformation, the vector is known in any other set of coordinate directions. In this book, vectors are denoted by placing an arrow above the symbol for the vector, such as $\vec{v}$.

Position, velocity, and acceleration. Position, velocity, and acceleration are all examples of vectors. If we consider a particle that has *position* $\vec{r}$ relative to some location, then the *velocity* of the particle is the time rate of change of its position

$$\vec{v} = d\vec{r}/dt, \tag{1.1}$$

where d/dt denotes the derivative with respect to time.* Similarly, the *acceleration* is the time rate of change of velocity

$$\vec{a} = d\vec{v}/dt. \tag{1.2}$$

Since statics is concerned with situations where $\vec{a} = \vec{0}$, our discussion of Eqs. (1.1) and (1.2) will be brief. If a particle's acceleration is zero, then integration of Eq. (1.2) shows the particle has constant velocity, which may be zero or nonzero. If the velocity is zero, then Eq. (1.1) shows the particle's position does not change, while if the velocity is nonzero but is constant, integration of Eq. (1.1) shows the particle's position changes as a linear function of time. If the acceleration is not zero, then the particle will move with velocity and position that change with time.

Newton's laws of motion

Inspired by the work of Galileo and others before him, Newton postulated his three laws of motion in 1687:

First Law. A particle remains at rest, or continues to move in a straight line with uniform velocity, if there is no unbalanced force acting on it.

Second Law. The acceleration of a particle is proportional to the resultant force acting on the particle and is in the direction of this force. The mathematical statement of this law† is

$$\boxed{\vec{F} = m\vec{a},} \tag{1.3}$$

where $\vec{F}$ is the resultant force acting on the particle, $\vec{a}$ is the acceleration of the particle, and the constant of proportionality is the mass of the particle m. In Eq. (1.3), $\vec{F}$ and $\vec{a}$ are vectors, meaning they have both size (or magnitude) and direction. Vectors are discussed in detail in Chapter 2.

Third Law. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

Newton's laws of motion, especially Eq. (1.3), are the basis of mechanics. They are postulates whose validity and accuracy have been borne out by countless experiments

Concept Alert

Newton's second law. Newton's second law, $\vec{F} = m\vec{a}$, is the most important fundamental principle upon which statics, dynamics, and mechanics in general are based.

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* Equations (1.1) and (1.2) are valid regardless of how a vector might be represented. However, the details of how the time derivative is evaluated depend on the particular vector representation (e.g., Cartesian, spherical, etc.) that is used. Dynamics explores these details further.

† Actually, Newton stated his second law in a more general form as $\vec{F} = d(m\vec{v})/dt$, where $\vec{v}$ is the velocity of the particle and $d(m\vec{v})/dt$ denotes the time rate of change of the product $m\vec{v}$, which is called the *momentum* of the particle. When mass is constant, this equation specializes to Eq. (1.3). For problems in which mass is not constant, such as in the motion of a rocket that burns a substantial mass of fuel, the more general form of Newton's second law is required.

and applications for more than three centuries. Unfortunately, there is no fundamental proof of their validity, and we must accept these as rules that nature follows. The first law was originally stated by Galileo. Of the three laws, only the second two are independent. In Eq. (1.3), we see that if the resultant force $\vec{F}$ acting on a particle is zero, then the acceleration of the particle is zero, and hence the particle may move with uniform velocity, which may be zero or nonzero in value. Hence, when there is no acceleration (i.e., $\vec{a} = \vec{0}$), the particle is said to be in *static equilibrium*, or simply *equilibrium*. The third law will play an important role when drawing free body diagrams, which we will see are an essential aid for applying $\vec{F} = m\vec{a}$.

1.5 Force

Forces are of obvious importance to us. In statics, we are usually interested in how structures support the forces that are applied to them, and how to design structures so they can accomplish the goal of supporting forces. In dynamics, we are usually interested in the motions of objects that are caused by forces that are applied to them. In this section, we discuss force in some detail, examine some different types of forces, and discuss how forces are produced.

Simply stated, a *force* is any agency that is capable of producing an acceleration of an unsupported body.* While this definition may seem vague, it is comprehensive. All forces are produced from the interaction of two or more bodies (or collections of matter), and the interaction between the bodies can take several forms, which gives rise to different ways that forces can be produced.

For many purposes, a force can be categorized as being either a *contact force* or a *field force*:

- **Contact force.** When two bodies touch, *contact forces* develop between them. In general, the contact forces are distributed over a finite area of contact, and hence, they are *distributed forces* with dimensions of force/area. If the bodies touch over only a small region, or if we replace the distributed force by an equivalent concentrated force as discussed in Chapter 7, then the contact forces are concentrated at a point. Contact forces are made up of two parts: a normal-direction force and a tangential-direction force, which is also called the *friction force*. Examples of contact forces include the forces between your feet and ground when you are standing, and the force applied by air to a building during a blowing wind.

- **Field force.** A force between bodies that acts through space is called a *field force*. Field forces act throughout the volume of an object and thus have dimensions of force/volume. Field forces are often called *body forces*. For many applications, we can represent a field force by a concentrated force that acts at a point. Examples of field forces include the weight of an object, the attractive force between the Earth and Moon, and the force of attraction between a magnet and an iron object.

Some examples of contact and field forces are shown in Fig. 1.5.

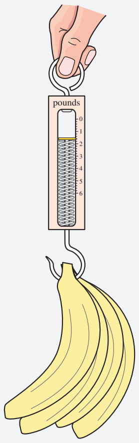
Although the preceding definition of contact forces is useful, more careful consideration of contact at an atomic length scale shows that a contact force is a special

* Whether or not a particular body does accelerate depends upon the combined action of *all* forces that are applied to the body.

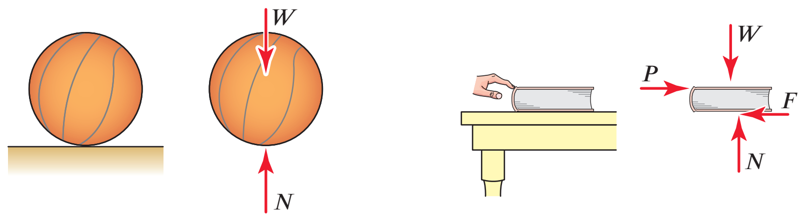
Interesting Fact

Measuring force. A force can cause an unsupported body to accelerate and also can cause a body (both unsupported and supported) to deform, or change shape. This suggests two ways to measure force. First, for an accelerating body with known mass m, by measuring the acceleration $\vec{a}$, we may then determine the force $\vec{F}$ applied to the body, using Newton's law $\vec{F} = m\vec{a}$. This approach is common in celestial mechanics and projectile motion, but it cannot be used for objects that are in static equilibrium.

A second approach that is more common for both static and dynamic applications is by measuring the *deformation* (i.e., shape change) that a force produces in an object whose behavior is known. An example is the handheld spring scale shown, which is being used to weigh bananas.



The weight of the bananas causes the spring's length to change, and because the spring's stiffness is known, the force the bananas apply to the scale can be determined. A brief historical discussion of mass and force measurements is given in J. C. Maxwell's notes on dynamics entitled *Matter and Motion*, Dover Publications, Inc., New York, 1991, the preface of which is dated 1877. A more contemporary discussion of force measurements (and measurements in general) is available from the *National Institute of Standards and Technology* (NIST) (see `http://www.nist.gov/`).



(a) (b)

Figure 1.5. Examples of contact forces and field forces. (a) A basketball rests on a hard level surface. (b) A book is pushed across a table with your finger. In both examples, the field force is the weight W of the object, and the contact forces are the normal force N, the friction force F, and the force P applied by your finger to the book. For the basketball, contact occurs over a very small region, and it is reasonable to idealize this as a point. For the book, contact occurs over the entire surface of the book cover, but it is nonetheless possible to model the contact forces by concentrated forces acting at a point.

case of a field force. As an atom from one surface comes very close to an atom on the opposite surface, the atoms never touch one another, but rather they develop a repulsive field force that increases rapidly as the two atoms come closer. However, the range of distances over which these forces act is very small (on the order of atomic dimensions), and for macroscopic applications, our definition of contact forces is useful.

1.6 Units and Unit Conversions

Units are an essential part of any quantifiable measure. Newton's law $F = ma$, written here in scalar form, provides for the formulation of a consistent and unambiguous system of units. We will employ both U.S. Customary units and SI units (International System*) as shown in Table 1.1.

Helpful Information

Dimensions versus units. *Dimensions* and *units* are different. A dimension is a measurable extent of some kind, while units are used to measure a dimension. For example, length and time are both dimensions, and meter and second, respectively, are units used to measure these dimensions.

Table 1.1. U.S. Customary and SI unit systems.

	System of Units	
Base Dimension	U.S. Customary	SI
force	pound (lb)	newton[a] (N) $\equiv$ kg·m/s^2
mass	slug[a] $\equiv$ lb·s^2/ft	kilogram (kg)
length	foot (ft)	meter (m)
time	second (s)	second (s)

[a] Derived unit.

Each system has three *base units* and a fourth *derived unit*. In the U.S. Customary system, the base units measure force, length, and time, using lb, ft, and s, respectively, and the derived unit is obtained from the equation $m = F/a$, which gives the mass unit as $\text{lb}\cdot\text{s}^2/\text{ft}$, which is defined as 1 *slug*. In the SI system, the base units measure mass, length, and time, using kg, m, and s, respectively, and the derived unit is obtained from the equation $F = ma$, which gives the force unit as $\text{kg}\cdot\text{m/s}^2$, which

* SI has been adopted as the abbreviation for the French *Le Système International d'Unités*.